How to Teach the Method of Unity

No. 615 Shelf........

TEACHERS' LIBRARY.

The Principal of the Appleton Street Grammar School shall have charge of this library. One Book may be taken out every Monday afternoon by each Teacher. Books may be renewed for one week.

Holyoke, Sept. 1884.

HOW TO TEACH

THE

METHOD OF UNITY.

(CODE 1883; SCHEDULE I, ARITHMETIC.)

An Exemplification of the Method, with its Practical Application to the Arithmetic of Standards IV to VII, and of Pupil-Teachers. Illustrated by Numerous Examples fully worked out, with Hints to Teachers, and Specimen Notes of Lessons.

BY

ALFONZO GARDINER,

THIRD EDITION, REVISED AND ENLARGED

JOHN HEYWOOD,
DEANSGATE, AND RIDGEFIELD, MANCHESTER;
AND 11, PATERNOSTER BUILDINGS,
LONDON.
1883.

WORKS BY ALFONZO GARDINER.

	s.	d.
Sound, Light, and Heat. 13th Ed., 204 pp.	1	6
Magnetism and Electricity (Extra Subject Series)	0	6

John Heywood's Complete Series of Home Lesson Books.

Book I.,	for Standard I.	Cloth,	4d.
„ II.	„ II.	„	5d.
„ III.	„ III.	„	6d.
„ IV.	„ IV.	„	8d.
„ V.	„ V.	„	9d.
„ VI.	„ VI.	„	9d.
„ VII.	„ VII.	„	preparing.

John Heywood's Parsing Tables. Mounted and Varnished, 60in. by 40in. 7 6

John Heywood's Analysis Table. Same size and price.

John Heywood's Parsing and Analysis Card. 8in. by 6in. 0 1

The Complete Standard Examination Guide 1 6

Gardiner's Inspectors' Test Cards—

Set A, Standards I. to VI. 1s. per packet.
Set B, „ II. to VII. preparing.

The Explanatory Geography. With Maps and Diagrams.

Part I.	for Standards	I. and II.	2d.
„ II.	„	III.	3d.
„ III.	„	IV.	4d.
„ IV.	„	V.	5d.
„ V.	„	VI. and VII.	5d.

How to Draw a Map. 6th Ed. 1 0

John Heywood, Manchester; and 11, Paternoster Buildings, London.

PREFACE TO FIRST EDITION.

AMONGST other great improvements the New Code of 1882 stipulates, in Schedule I, Arithmetic, Standard V, that the teaching of Rule of Three is to be by "The Method of Unity."

Thoughtful teachers have for a long time been dissatisfied with the ordinary method of "statement." It is true *results* are arrived at by its use, and children of fair ability have had no difficulty in attaining a certain *mechanical accuracy*, but without any thorough knowledge of the method by which the results have been obtained. The great aim in the teaching of Arithmetic is not so much to provide the children with a certain number of mechanical devices by which special questions are to be worked, as to improve the *reasoning powers*. The ordinary method of "statement" fails to do this, as the theory of Ratio and Proportion is far beyond the capacity of children, and requires, for its thorough comprehension, a fairly mature and well trained mind.

The Method of Unity, or working by "First Principles," as it is often called, is simplicity itself. It can be easily understood by even a dull child, and it is scientifically accurate. One reason why it has not been more used is on account of the difficulty children find in *arranging* the work so as to make neat and tidy papers for the Inspector's keen eye. This difficulty is here met by one or two simple plans of arrangement, which the author has used for the last ten years with unvarying success. As to the method itself, teachers who already know it will find little that is new, excepting the plan of arrangement. The application of the method to Simple and Compound Proportion, Interest, Percentages, Discount, and Stocks (see Schedule I, Standards V to VII) here exemplified will be sufficient to show to the practical teacher the wide range over which the method may be applied, and the manner of treating simple as well as more complicated problems. Nearly all the exercises in "John Heywood's Complete Series of Home Lesson Books," in the above mentioned rules (together with numerous other typical examples), are worked out in full.

ALF. GARDINER.

Little Holbeck Board School, Leeds,
July, 1882.

PREFACE TO THIRD EDITION.

THE rapid sale, in less than a year, of two editions of this little work shows that it has met a want.

Advantage has been taken of the issue of a third edition to considerably enlarge the book. Appendix I. contains solution of all the questions (to which the method is applicable) in "John Heywood's Complete Series of Home Lesson Books," Standards V. and VI., which are not worked out in the body of the work. References by number are made in the answer books to the various questions.

Appendix II. gives very neat and concise directions for *stating* Rule of Three, single and double. They were drawn up some years since by the Rev. J. W. W. Drew, of St. Edward's, Romford, and are so clear and distinct, and presented in such a handy form, that young teachers will no doubt find them a great help with dull children. To pupil-teachers drawing up notes of lessons they supply just what is wanted to show the train of reasoning. Our best thanks are due for readily granted permission to make use of them, and though they give little help towards understanding the "Method of Unity," no apology is necessary for their introduction, since the *Instructions to Inspectors* state that "if the answers are correct, and have been intelligently worked by either method, you will, of course, accept them."

The early part of the book has been made much clearer, and in the body of the work a number of more complex questions have been incorporated (see Nos. 64, 65, 66, 67, 68, 69, 93, 94, 108, 123, 124, and 134). Several typical "Stock sums" are added to this section of the book (see Nos. 142, 143, 145, 148, 149).

It has been objected against the method of arranging Double Rule of Three here presented, that the work loses its educational value, and becomes a merely mechanical exercise. This objection is groundless, as exactly the same train of reasoning is gone through in forming the final fraction only, as in building up several fractions line by line. In Appendix I. the examples are alternately worked by the long and the short methods, so that teachers who prefer the former may have it fully illustrated. ALF. GARDINER.

Little Holbeck Board School, Leeds,
June, 1883.

TABLE OF CONTENTS.

CONTENTS.

HOW TO TEACH

THE METHOD OF UNITY.

1. The Method of Unity is frequently called working by "*First Principles*," because a knowledge of "the first principles," or fundamental rules of arithmetic alone is sufficient for the solution of simple problems, and every problem, no matter how complicated its appearance, reduces itself finally to cases of multiplication and division. It is also known as the "*French method*," because great use is made of it in the teaching of elementary arithmetic in French schools.

2. Explanation of Terms.—All arithmetical questions, to which this method of working is applicable, however complicated their form, will be found to consist of *two parts*—

(1) the *statement*, which is often a *supposition*,

(2) the *demand*, or question to be solved.

3. Before proceeding to find the thing demanded, the statement, or supposition, is *reduced to unity*—hence the name, "The Method of Unity, or The Unitary Method," applied to this particular manner of working out various forms of arithmetical problems included under the comprehensive terms of Ratio and Proportion. Many others, which range themselves under no particular rule, can also be worked out by the same method.

4. Advantages of the Method.

(i.)—It is a simple and logical way of working all problems in Proportion.

(ii.)—It exercises the reasoning powers, and is therefore more useful than the ordinary method of "statement," which enables a problem to be worked without the pupil knowing the "why and wherefore."

(iii.)—It is of wide application. By a little care in making the "statement line" every problem in Simple and Compound Proportion, Interest, Percentages, Discount, and Stocks may be easily and intelligently worked.

5. Preliminary Practice. — Before the "Unitary Method" can be explained satisfactorily to the children, they must have had a considerable amount of practice in the reasoning out of such simple arithmetical problems as the following, which may form easy exercises in the arithmetic of Standards II. and III.

(Ex. 1.)—If four men earn 16 shillings as their day's wages, how much will one man earn?

(1) Divide the problem into its two parts—*

(*a*) *statement*—4 men earn 16s.,

(*b*) *demand*—how much will one man earn?

(2) Consider which of the terms in the *statement* is of the same kind as the *demand*. Your answer requires to be like that term. In this case it is shillings; write down the statement so that the last term mentioned in it is of the same denomination (*i.e.*, the same kind of term) as the answer you require, *viz.*, shillings, thus—

4 men earn 16 shillings.

* In drawing up Notes of Lessons the steps of the reasoning must be shown in this way. See pp. 106–7.

The answer will always be of the same kind as the last term in this arranged statement.

(3) Reduce the first term in this arranged statement to unity, reasoning thus—1 man will earn four times less than 4 men, therefore—

1 man will earn 16s. divided by 4.

(4) Work out the division—

$$4 \underline{\backslash 16 \text{ shillings}}$$
$$\underline{\underline{4 \text{ shillings.}}} \quad \textit{Ans.}$$

(Ex. 2.)—A sack of flour is eaten in 27 days by a family of nine persons, how long will it last one person?

(1) The two parts of the problem are—

(*a*) *statement*—a sack of flour is eaten in 27 days by 9 persons.

(*b*) *demand*—how long will it last 1 person?

(2) Arrange the statement so that the last term is of the same kind as the required answer, *viz.*, days, thus—

9 persons eat a sack of flour in 27 days.

(3) Reduce the first term in this arranged statement to unity, reasoning thus—1 person will be 9 *times longer* in eating the sack of flour than 9 persons, therefore—

1 person will eat a sack of flour in 27 days multiplied by 9.

(4) Perform the multiplication—

$$\begin{array}{l} 27 \text{ days} \\ \underline{\ \ 9} \\ \underline{\underline{243}} \text{ days. } \textit{Ans.} \end{array}$$

(Ex. 3.)—A guinea buys six yards of cloth, how much is that per yard?

(1) The two parts of the problem are—

(*a*) *statement*—21 shillings buy 6yds. of cloth.

(*b*) *demand*—price of 1 yard ?

(2) Arrange the statement so that the last term is of the same kind as the required answer, *viz.*, money, thus—

6 yards of cloth cost 21 shillings.

(3) Reduce the first term in this arranged statement to unity, reasoning thus—1 yard will cost 6 times less than 6 yards, therefore—

1 yard will cost 21/- divided by 6.

(4) Perform the compound division—

```
     s.  d.
  6 |21  0
    ------
     3   6 per yard.   Ans.
```

(Ex. 4.)—The poor rate is 2/6 in the pound. How much must a person pay whose house is rated at £21 ?

(1) The two parts of the problem are—

(*a*) *statement*—poor rate is 2/6 in the pound.

(*b*) *demand*—amount to be paid on £21 ?

(2) Arrange the statement so that the last term is of the same kind as the required answer, *viz.*, so much rate, thus—

Amount paid on £1 is 2/6.

(3) As the first term in this arranged statement is already in terms of unity, nothing more requires to be done with it; but the demand requires us to find how much must be paid on £21, reasoning thus—amount to be paid on £21 will be 21 times that on £1, therefore—

Amount paid on £21 is 21 times 2/6.

(4) Performing the multiplication—

```
       s. d.
        2  6
           7×3=21
      ———————
       17  6
           3
     ————————
    £2 12  6  Ans.
```

(Ex. 5.)—How much can a bankrupt pay in the pound if his assets (that is, the value of his property) are £1,260, and he owes £1,575?

(1) The two parts of the problem are—

(*a*) *statement*—assets worth £1,260 pay debts of £1,575.

(*b*) *demand*—how much of every pound is paid?

(2) Arrange the statement so that the last term is of the same kind as the required answer, *viz.*, assets, because, as he pays debts of £1,575 with only £1,260, he must pay £1 with a certain portion of the £1,260, thus—

Payment of £1,575 is made with £1,260.

(3) Reduce the first term of this arranged statement to unity, reasoning thus—£1 will be paid with 1,575 times less than £1,575, therefore—

£1 will be paid with £1,260 divided by 1,575.

(4) Performing the division—

```
          £
        1260
          20
1575 \25200\ 16 shillings.  Ans.
       1575
       ————
        9450
        9450
        ....
```

6. It will be seen that the whole of these examples require only multiplication or division for their solution. In actual work, on slate or paper, however, it will be manifest that the reasoning out and *writing down* of each step would be very tedious, and that an immense amount of time would have to be devoted to arithmetic, for the children to have sufficient practice to ensure thoroughness, accuracy, and neatness.

7. Shortening of Work.—The method of reasoning out must in all cases be followed, but the work may be much shortened, as regards the writing, in two ways :—

(1) by the use of arithmetical signs;

(2) by concise forms of statement.

8. Use of Arithmetical Signs.—The ordinary sign for multiplication (×) is so convenient that children, from Standard II. upwards, ought to know its use thoroughly. The same may be said of the sign for division (÷) ; but it will be found more useful in practice to express division by writing the divisor *under* the dividend, separating them by means of a horizontal line, so as to form the terms into a *vulgar fraction.*

9. Since it will generally be more advantageous to teach "Rule of Three by the Method of Unity" after the children have gone through the elementary course of fractions now prescribed for Standard V., this method of forming the two terms into a fraction, or, in other words, showing that the upper number (the numerator of the fraction) is to be divided by the lower one (the denominator of the fraction), will not be difficult to understand, especially if the children comprehend the following **definition of a fraction** :—

A fraction is formed by dividing a unit into a number of equal parts. One or more of these parts is taken to form the upper term of the

fraction, called the numerator. The number of parts into which the unit is divided is called the denominator. It is placed under the numerator, being separated from it by a horizontal line.

10. The meaning of the sign for equality (=, equals) should also be thoroughly understood.

11. "Cancelling" may also be explained in the working out of these simple problems, but it is better to delay its use for a short time (see par. **15**, p. 15.)

12. Concise Forms of Statement.—The success or failure of this method will depend upon the way in which the *arranged statement* is written. If children cannot do this *well*, the "Method of Unity," instead of simplifying the working of arithmetical problems, makes it more difficult. The *statement* must be in such a form that—

(1) as few words as possible are used. Two or three will generally be enough, and the word (or words) denoting the kind of answer (*i.e.*, the thing demanded) will generally be the *first* one in the statement line.

(2) the particular form of words used must be in such grammatical relation that, neither as a whole, nor (except rarely) in part, will they have to be varied throughout the steps of the method;

(3) all the wording, must, as much as possible, be placed *before the first term.*

(4) the terms of the statement line must be arranged, in every case, with the sign for equals (=) between them.

13. In actual school work problems 1 to 5 would assume the following forms, when the foregoing suggestions for statement, &c., are applied to them.

(Ex. 1.)—If four men earn 16 shillings as their day's wages, how much will one man earn?

$$\text{Wages earned by 4 men} = 16 \text{ shillings}$$
$$\text{,, ,, 1 man} = \frac{16}{4} \text{ ,,}$$
$$\frac{16}{4} = 4 \text{ shillings. } \textit{Ans.}$$

(Ex. 2.)—A sack of flour is eaten in 27 days by a family of nine persons, how long will it last one person?

$$\text{Time for 9 persons} = 27 \text{ days}$$
$$\text{,, ,, 1 person} = 27 \times 9 \text{ days}$$
$$27 \times 9 = 243 \text{ days. } \textit{Ans.}$$

(Ex. 3.)—A guinea buys six yards of cloth, how much is that per yard?

$$\text{Price of 6 yards} = 21 \text{ shillings}$$
$$\text{,, ,, 1 yard} = \frac{21}{6} \text{ ,,}$$
$$\frac{21}{6} = 3/6. \ \textit{Ans.}$$

(Ex. 4.)—The poor rate is 2/6 in the pound. How much must a person pay whose house is rated at £21?

$$\text{Rate paid on £1} = 2/6$$
$$\text{,, ,, ,, £21} = 2/6 \times 21$$
$$2/6 \times 21 = \text{£2 12s. 6d. } \textit{Ans.}$$

(Ex. 5.)—How much can a bankrupt pay in the pound if his assets are £1,260, and he owes £1,575?

$$\text{Sum paid instead of £1575} = \text{£1260}$$
$$\text{,, ,, ,, £1} = \frac{\text{£1260}}{1575}$$
$$\frac{\text{£1260}}{1575} = \frac{\text{£1260} \times 20}{1575} = \frac{25200}{1575} \text{ shillings} = 16/-. \ \textit{Ans.}$$

14. In general no other plan than the one here illustrated must be allowed. *This method of stating must*

be stereotyped, as it were, and never departed from. Children will then get to make their statements quickly, correctly, and neatly. It is not advisable, as a rule, to allow the work to be written down as below, though the train of reasoning may be gone through in this form :—

(1) 4 men earn 16 shillings

∴ 1 man earns $\frac{16}{4}$ „ $= 4/-$ *Ans.*

(2) 9 persons eat a sack of flour in 27 days

∴ 1 person eats „ $27 \times 9 = 27$ dys.

(3) 6 yards of cloth are bought for 21 shillings

∴ 1 yard „ is „ $\frac{21}{6}$ „ $= 3/6$. *Ans.*

15. Cancelling.—The pupils should now be thoroughly initiated into the mysteries of cancelling, if they do not already understand them. The theory of the process is very simple, and follows from the second of the two following propositions :—

(1) The value of a fraction is not altered if both the numerator and denominator be *multiplied* by the same number.

(2) The value of a fraction is not altered if both the numerator and denominator be *divided* by the same number.

16. The greatest difficulty to be overcome is to prevent the children dividing two terms in the numerator, or two in the denominator, by some common factor, instead of *one* in the numerator with *one* in the denominator. Numerous exercises in the reduction of *compound fractions* should be carefully worked out on the blackboard. Several lessons may be advantageously occupied with this work alone, as nothing but long continued practice can make children expert in the finding out of common factors.

17. Common Factors.—The following rules may be gradually taught. They will help children to find common factors, and so ensure accuracy and thoroughness in cancelling :—

A number is exactly divisible by—

2 when its *last* figure is 0, or an even digit, as 328.
3 „ the *sum* of its digits is divisible by 3, as 732.
4 „ its *last two* figures are divisible by 4, as 2576.
5 „ its *last* figure is 5 or 0.
6 „ its *last* figure is an even digit, and the number is also divisible by 3, as 5976.
8 „ its *last three* figures are divisible by 8, as 3640.
9 „ the *sum* of its digits is divisible by 9, as 27405.
10 „ its *last* figure is 0.

The factors in black type should be first taught.

18. It may also be noticed that—

(*a*) A number is divisible by 7, 11, or 13 when it is composed of sets of *three* figures repeated in the same order, as 642642, 25025 (=025025).

(*b*) A number is divisible by 37 when it is composed of digits repeated *three* times, or any multiple of three times, as 111, 333, 555555.

(*c*) A number is divisible by 73 and 137 when it is composed of *four* figures repeated in the same order, as 35413541, 7620762 (=07620762).

19. Rule of Three, or Simple Proportion.—All Simple Rule of Three sums resolve themselves into *two parts*, as in the easy problems we have been considering. These two parts include *three* terms and require a *fourth* one to be found.

(*a*) *statement*—consisting of *two* terms, *both* of which are given.

(*b*) *demand*—also consisting of *two* terms, *one only of which is given*; the other, corresponding to the second of the given terms in the statement, has to be found.

In all cases a plan, similar to the one already explained, is pursued, but *three* steps are now required.

(1) 1st step—The *statement* containing the two given terms is arranged so that the last of these two terms is of the same kind as the required answer (Ex. 1).

(2) 2nd step—The *first term*, in this arranged statement, is reduced to unity.

(3) 3rd step—The *given term* in the *demand* is now introduced, in place of unity.

20. The following examples illustrate the application of the method to various kinds of problems. It will be noticed that—

(1) the statement in the first step is as concise as possible (par. **12** (1), p. 13);

(2) nearly all the wording is placed *before* the first term of the first step, and little grammatical alteration is afterwards made in it (par. **12** (3), p. 13).

(Ex. 6.)—If 4 men earn 16 shillings, how much will 13 men earn?

As the answer must be money the statement will be—

1st *step or Statement line*—Wages earned by 4 men = 16 shillings.

Reducing this statement of the number of men to unity, since 1 man will earn 4 times less than 4 men, we get—

2nd step or Unity line—Wages earned by 1 man $=\frac{16}{4}$ shillings.

We have now to find the wages earned by 13 men, and since the second step, if divided out, would give us the amount earned by *one* man, 13 times one man's wages will be the required answer. In practice, however, the second step is *not worked out*, but is left as above, and the given term of the third step is introduced into the numerator or denominator of the fraction, as the reasoning indicates, and a compound fraction is thus formed which is afterwards reduced.

3rd step or Demand line—Wages earned by 13 men $=\frac{16\times 13}{4}$ shillings.

Reducing this fraction we get the answer, thus—

$$\frac{\overset{4}{\cancel{16}}\times 13}{\cancel{4}}=4\times 13=52s.=£2\ 12s.\quad \textit{Ans.}$$

If neat and tidy work is required (and what teacher does not aim at it?) never reduce the fraction formed by the third step at the place in the third line, where the terms composing it are arranged. Set it down again at a suitable distance *below* the last step of the reasoning, and then cancel, or otherwise reduce it. In other words follow the plan *exactly* as shown in these examples.

21. Arrangement of Work in Simple Proportion. On the scholar's slate, exercise book, or examination paper the above problem will appear, thus—

Wages earned by 4 men = 16 shillings.

" " 1 " $=\frac{16}{4}$ "

" " 13 " $=\frac{16\times 13}{4}$ "

$$\frac{\overset{4}{\cancel{16}}\times 13}{\cancel{4}}=4\times 13=52s.=£2\ 12s.\quad \textit{Ans.}$$

(Ex. 7.)—If 20lbs. of sugar cost 4s., how much will one cwt. cost?

As the answer must be money, 4s. will be the *last* term in the statement.

Reduce 1 cwt. to the same denomination as the first term in the statement, *i.e.*, 1cwt. = 112lbs.

1st step or Statement line—Cost of 20lbs. = 4 shillings.

2nd step or Unity line— „ 1lb. $= \frac{4}{20}$ „ { 1lb. will cost 20 times less than 20lbs.

3rd step or Demand line— „ 112lbs. $= \frac{4 \times 112}{20}$ „

4th step $\frac{\cancel{4} \times 112}{\cancel{20}_{5}}$s. $= \frac{112}{5}$s. $= 22$s. $4\frac{3}{4} + \frac{1}{5}$d. $=$ £1 2s. $4\frac{3}{4}$d. $+ \frac{1}{5}$. *Ans.*

Or = 22s. $4\frac{4}{5}$d. = £1 2s. $4\frac{4}{5}$d. *Ans.*

The working out of the fraction $\frac{112}{5}$ may either be done in the margin of the slate, exercise book, or examination paper, or *underneath* the 4th line, thus—

5	112 shillings
20	22s. $4\frac{3}{4} + \frac{1}{5}$d.
	£1 2s. $4\frac{3}{4}$d. $+ \frac{1}{5}$.

On slate I generally have *all* the working done as just shown, but on an examination paper the final reduction of the fraction is done in the margin which is ruled off, on the right hand side of the paper, for that purpose.

(Ex. 8.)—If 10 dozen herrings cost 5/-, what will 5 dozen cost?

$$\begin{array}{ll} & \text{Doz. Shillings.} \\ \text{Cost of} & 10=5 \\ \quad ,, & 1=\frac{5}{10} \\ \quad ,, & 5=\frac{5\times 5}{10} \end{array}$$

$$\frac{\overset{1}{\cancel{5}}\times 5}{\underset{2}{\cancel{10}}}=\frac{5}{2}=2/6. \quad \textit{Ans.}$$

It is immaterial whether the statement line be written as in this example, with doz. and shillings *above* the figures, or *at the side*, thus—

Cost of 10 doz. = 5 shillings.

The first way generally makes the neatest work if children are taught to leave at least a couple of lines between every sum.

(Ex. 9.)—A man works 6 days for 48/-, how long will he work for £4?

£4 = 4 × 20/- = 80/-

$$\begin{array}{ll} & \text{Shillings. Days.} \\ \text{Time for} & 48=6 \\ \quad ,, & 1=\frac{6}{48} \\ \quad ,, & 80=\frac{6\times 80}{48} \end{array}$$

$$\frac{\cancel{6}\times \overset{10}{\cancel{80}}}{\underset{\cancel{8}}{\cancel{48}}}=10 \text{ days.} \quad \textit{Ans.}$$

(Ex. 10.)—What will be the cost of 30 ducks if 4 couples cost £1?

4 couples = 8 ; £1 = 20/-

$$\begin{array}{rl} & \text{Ducks. Shillings.} \\ \text{Cost of } 8 & = 20 \\ \text{,,} \quad 1 & = \frac{20}{8} \\ \text{,,} \quad 30 & = \frac{20 \times 30}{8} \end{array}$$

$$\frac{\overset{5}{\cancel{20}} \times \overset{15}{\cancel{30}}}{\overset{\cancel{8}}{\cancel{2}}} = 15 \times 5 = 75/\text{-} = £3\ 15\text{s.} \quad \textit{Ans.}$$

(Ex. 11.)—If 15 men can do a piece of work in 15 hours, how many men must be employed to do it in 5 hours?

$$\begin{array}{rl} & \text{Hours. Men.} \\ \text{No. of men for } 15 & = 15 \\ \text{,,} \quad 1 & = 15 \times 15 \\ \text{,,} \quad 5 & = \frac{15 \times 15}{5} \end{array}$$

$$\frac{\overset{3}{\cancel{15}} \times 15}{\cancel{5}} = 15 \times 3 = 45 \text{ men.} \quad \textit{Ans.}$$

Note the reasoning in the *second* step. If work, which it takes 15 men 15 hours to do, has to be done in 1 hour, 15 times as many men must be employed to do it. Careful teaching, *i.e., causing every one of the three steps to be properly reasoned out,* will alone ensure children placing the terms in their proper position. Careless children always form the second term in the unity line by division unless they are made to reason it out in every instance. Cases like this are easy of demonstration, and if worked out by the old method of "stating" would present no difficulty to the children, but they are the cause of frequent mistakes, through mere carelessness, when worked out by the Unitary Method.

(Ex. 12.)—If I bought 12 yards of silk for £3 13s. $3\frac{1}{2}$d., how many yards could I have bought for a ten pound note?

£3 13s. $3\frac{1}{2}$d.	£10
20	20
73	200
12	12
879	2400
2	2
1759 halfpence	4800 halfpence.

$$\begin{array}{lrl} & \text{Halfpence.} & \text{Yards.} \\ \text{No. of yards for} & 1759 = & 12 \\ \quad " & 1 = & \frac{12}{1759} \\ \quad " & 4800 = & \frac{12 \times 4800}{1759} \end{array}$$

$\frac{12 \times 4800}{1759} = \frac{57600}{1759} = 32\frac{1312}{1759}$yds. = 32yds. 2qrs. 3nls. 2in. + $178\frac{3}{4}$. *Ans.*

(Ex. 13.)—If 30 men do a piece of work in ten days—(*a*) How many days will 20 men be in doing it? (*b*) How many men could do it in 3 days? (*c*) How many days could 40 men do it in?

There are here *three* separate questions. Notice the reasoning in the unity line in all the three cases.

$$\begin{array}{llrl} & & \text{Men.} & \text{Days.} \\ (a) & \text{No. of days for} & 30 = & 10 \\ & \quad " & 1 = & 10 \times 30 \\ & \quad " & 20 = & \frac{10 \times 30}{20} \end{array}$$

$$\frac{\cancel{10} \times \overset{15}{\cancel{30}}}{\underset{2}{\cancel{20}}} = 15 \text{ days. } \textit{Ans.}$$

$$\begin{array}{llrl} & & \text{Days.} & \text{Men.} \\ (b) & \text{No. of men for} & 30 = & 10 \\ & \quad " & 1 = & 30 \times 10 \\ & \quad " & 3 = & \frac{30 \times 10}{3} \end{array}$$

$$\frac{\overset{10}{\cancel{30}} \times 10}{\cancel{3}} = 10 \times 10 = 100 \text{ men. } \textit{Ans.}$$

$$\text{(c) No. of days for } \overset{\text{Men.}}{30} = \overset{\text{Days.}}{10}$$

$$\text{,, } 1 = 10 \times 30$$

$$\text{,, } 40 = \frac{10 \times 30}{40}$$

$$\frac{\cancel{10} \times 30}{\underset{4}{\cancel{40}}} = \frac{30}{4} = 7\tfrac{2}{4} = 7\tfrac{1}{2} \text{ days. } \textit{Ans.}$$

(Ex. 14.)—If 1 ton 4 cwt. can be carried from Manchester to Liverpool for £2 13s., what weight can be carried for £5 11s. 6¼d.?

£2 13s. = 2544 farthings. £5 11s. 6¼d. = 5353 farthings.

1 ton 4 cwt. = 24 cwt.

$$\text{Weight for } \overset{\text{Farthings.}}{2544} = \overset{\text{Cwt.}}{24}$$

$$\text{,, ,, } 1 = \frac{24}{2544}$$

$$\text{,, ,, } 5353 = \frac{24 \times 5353}{2544}$$

$$\frac{\overset{2}{\cancel{24}} \times 5353}{\underset{\underset{106}{\cancel{212}}}{\cancel{2544}}} = \frac{5353}{106} = 50\tfrac{53}{106} \text{ cwt.} = 50\tfrac{1}{2} \text{ cwt.}$$

$$= 2 \text{ tons } 10 \text{ cwt. } 2 \text{ qrs. } \textit{Ans.}$$

(Ex. 15.)—If two pounds of sugar cost the same as one pound of cheese, how many pounds of cheese are worth nine pounds of sugar?

The statement requires special attention.

$$\text{Pounds of cheese worth } \overset{\text{lb.}}{2} \text{ of sugar} = \overset{\text{lb.}}{1}$$

$$\text{,, ,, ,, } 1 \text{ ,, } = \frac{1}{2}$$

$$\text{,, ,, ,, } 9 \text{ ,, } = \frac{1 \times 9}{2}$$

$$\frac{1 \times 9}{2} = 4\tfrac{1}{2}\text{lb. cheese. } \textit{Ans.}$$

(Ex. 16.)—How much will 16/6 in silver weigh if 1lb. troy be worth 66/-?

1lb. troy = 12oz. 16/6 = 33 sixpences. 66/- = 132 sixpences.

$$\begin{array}{ll} & \text{s.} \quad \text{oz.} \\ \text{Weight of } 132 & = 12 \\ \quad ,, \quad 1 & = \dfrac{12}{132} \\ \quad ,, \quad 33 & = \dfrac{12 \times 33}{132} \end{array}$$

$$\frac{\overset{3}{\cancel{12}} \times \cancel{33}}{\underset{\cancel{4}}{\cancel{132}}} = 3\text{oz.} \quad \textit{Ans.}$$

(Ex. 17.)—I lent John half-a-crown for three weeks, how long ought he to lend me two shillings?

The unity line requires special attention, for it is manifest, on consideration, that you will lend one penny for a longer time than you will lend 30 pence, if other conditions are the same.

2/6 = 30 pence. 2/- = 24 pence.

$$\begin{array}{ll} & \text{Pence. Weeks.} \\ \text{Time for } 30 & = 3 \\ \quad ,, \quad 1 & = 3 \times 30 \\ \quad ,, \quad 24 & = \dfrac{3 \times 30}{24} \end{array}$$

$$\frac{\cancel{3} \times 30}{\underset{8}{\cancel{24}}} = \frac{30}{8} = 3\tfrac{6}{8} = 3\tfrac{3}{4} \text{ weeks.} \quad \textit{Ans.}$$

(Ex. 18.)—A man bought 100 oranges at 2 a penny and 50 more at 1d. each. He sold the lot at 3 for 2d.; did he gain or lose, and how much?

First find cost of all the oranges.

Cost of	100 oranges	@ 2 a penny	= 50 pence.
,,	50 ,,	@ a penny each	= 50 ,,
,,	150 oranges		= 100 pence.

Then find selling price.

$$\begin{array}{rl} & \text{Oranges. Pence.} \\ \text{Price for } 3 & = 2 \\ \text{,, } \quad 1 & = \frac{2}{3} \\ \text{,, } \quad 150 & = \frac{2 \times 150}{3} \end{array}$$

$$\frac{2 \times \cancel{150}^{50}}{\cancel{3}} = 2 \times 50 = 100 \text{ pence selling price.}$$

He therefore neither gained nor lost. *Ans.*

(Ex. 19.)—There are provisions in a town sufficient to support 4,000 soldiers for 3 months. How many men must be sent away in order to make these provisions last for 8 months?

$$\begin{array}{rl} & \text{Months. Soldiers.} \\ \text{Soldiers supported for } 3 & = 4000 \\ \text{,, } \quad \text{,, } \quad 1 & = 4000 \times 3 \\ \text{,, } \quad \text{,, } \quad 8 & = \frac{4000 \times 3}{8} \end{array}$$

$$\frac{\cancel{4000}^{500} \times 3}{\cancel{8}} = 500 \times 3 = 1500 \text{ soldiers.}$$

$\therefore$ number to be sent away $= 4000 - 1500 = 2500$. *Ans.*

(Ex. 20.)—How many loaves at 4½d. each are equal to 30 at 9d. each?

Children will probably find a difficulty in making a neat statement, and they are almost certain to make a mistake in reducing to unity. The two parts of the problem are—

(*a*) *statement*—30 loaves at 9 pence each,

(*b*) *demand*—number of loaves at 4½d. each, of a total equal value. As each loaf in the demand is of less value than in the statement, there must be *more* of them to be worth the same amount of mone-

9d. × 2 = 18 halfpence. $4\frac{1}{2}$d. × 2 = 9 halfpence.

	Halfpence.		Loaves.
Number of loaves at	18	each	= 30
" "	1	"	$= 30 \times 18$
" "	9	"	$= \frac{30 \times 18}{9}$

$$\frac{30 \times \overset{2}{\cancel{18}}}{\cancel{9}} = 30 \times 2 = 60 \text{ loaves. } \textit{Ans.}$$

The question may be worked more neatly thus—

	Pence.		Loaves.
Number of loaves at	9	each	= 30
" "	1	"	$= 30 \times 9$
" "	$4\frac{1}{2}$	"	$= \frac{30 \times 9}{4\frac{1}{2}}$

$$\frac{30 \times \overset{2}{\cancel{9}}}{\cancel{4\frac{1}{2}}} = 30 \times 2 = 60 \text{ loaves. } \textit{Ans.}$$

22. Complex Fractions.—Though the Code requirements of Standard V. do not specify a knowledge of *complex fractions*, the work will be much facilitated if children know a simple method of reducing them. Thus, Example 20 might be worked thus, though, in this particular case, the method of reduction just shown is the best—

$$\frac{30 \times 9}{4\frac{1}{2}} = \frac{\frac{30}{1} \times \frac{9}{1}}{\frac{9}{2}} = \frac{30 \times \cancel{9} \times 2}{\cancel{9}} = 30 \times 2 = 60. \quad \textit{Ans.}$$

23. In simplifying a complex fraction, experience shows the following plan to be the best. It necessitates a few more figures than by other methods, but it *ensures accuracy*, and accuracy is the first thing to be aimed at, brevity follows after :—

(1) Reduce both whole and mixed numbers to improper fractions.

(2) Retain these fractions in their proper position as numerator or denominator.

(3) Arrange all the "extremes" (or outside numbers) for a new numerator, and then all the "means" (or middle numbers) for a new denominator.

(4) Cancel and multiply.

(Ex. 21.)—Simplify $\dfrac{18 \times 3\frac{1}{2}}{15\frac{3}{4}}$.

(1) Reduce each of the terms in both numerator and denominator to improper fractions, retaining them, *in these fractional forms*, as numerator and denominator respectively.

$$\dfrac{\dfrac{18}{1} \times \dfrac{7}{2}}{\dfrac{63}{4}}$$

(2) Arrange all the extremes for a new numerator and the means for a new denominator, thus—

$$\left.\begin{array}{ll} \text{Extremes......} & 18 \times 7 \\ \text{Means ....} & \;\,1 \quad\;\; 2 \\ \hline \text{Mean ....} & \;\,63 \\ \text{Extreme ......} & \;\;\,4 \end{array}\right) = \dfrac{18 \times 7 \times 4}{2 \times 63} \begin{array}{l} \text{extremes.} \\ \text{means.} \end{array}$$

(3) Cancel and multiply (if necessary)—

$$\dfrac{\overset{\cancel{2}}{\cancel{18}} \times \cancel{7} \times 4}{\cancel{2} \times \underset{\cancel{9}}{\cancel{63}}} = 4. \quad \textit{Ans.}$$

On the child's slate or paper the work appears thus—

$$\dfrac{18 \times 3\frac{1}{2}}{15\frac{3}{4}} = \dfrac{\dfrac{18}{1} \times \dfrac{7}{2}}{\dfrac{63}{4}} = \dfrac{\overset{\cancel{2}}{\cancel{18}} \times \cancel{7} \times 4}{\underset{\cancel{9}}{\cancel{63}} \times \cancel{2}} = 4. \quad \textit{Ans.}$$

(Ex. 22.)—Simplify $\dfrac{1}{2\frac{1}{3} \times 4\frac{1}{3}}$.

$$\frac{1}{2\frac{1}{3} \times 4\frac{1}{3}} = \frac{\frac{1}{1}}{\frac{7}{3} \times \frac{13}{3}} = \frac{3 \times 3}{7 \times 13} = \frac{9}{91}. \quad \textit{Ans.}$$

(Ex. 23.)—Simplify $\dfrac{16\frac{4}{5}}{12 \times 7}$.

$$\frac{16\frac{4}{5}}{12 \times 7} = \frac{\frac{84}{5}}{\frac{12}{1} \times \frac{7}{1}} = \frac{\overset{7}{\cancel{84}}}{\cancel{12} \times \cancel{7} \times 5} = \frac{1}{5}. \quad \textit{Ans.}$$

(Ex. 24.)—Simplify $\dfrac{2\frac{1}{4} \times 3}{5\frac{1}{5}}$.

$$\frac{2\frac{1}{4} \times 3}{5\frac{1}{5}} = \frac{\frac{9}{4} \times \frac{3}{1}}{\frac{26}{5}} = \frac{9 \times 3 \times 5}{4 \times 26} = \frac{135}{104} = 1\tfrac{31}{104}. \quad \textit{Ans.}$$

This method of reduction will now be used, when necessary, in resolving the fraction formed at the third step.

(Ex. 25.)—How long will a person be in saving £3 if he puts by 1/6 per week?

$$1/6 = 18\text{d.} \quad £3 \times 20 \times 12 = 720\text{d.}$$

	Pence.	Weeks.
Time to save	18 =	1
„	1 =	$\frac{1}{18}$
„	720 =	$\frac{1 \times 720}{18}$

$$\frac{1 \times \overset{40}{\cancel{720}}}{\cancel{18}} = 40 \text{ weeks.} \quad \textit{Ans.}$$

Or, $1/6 = 1\frac{1}{2}/\text{-}$. $£3 = 60/\text{-}$

$$\begin{array}{lrl} & \text{s.} & \text{Weeks.} \\ \text{Time to save} & 1\frac{1}{2} & = 1 \\ \quad ,, & 1 & = \dfrac{1}{1\frac{1}{2}} \\ \quad ,, & 60 & = \dfrac{1 \times 60}{1\frac{1}{2}} \end{array}$$

$$\frac{1 \times 60}{1\frac{1}{2}} = \frac{\frac{1}{1} \times \frac{60}{1}}{\frac{3}{2}} = \frac{\overset{20}{\cancel{60}} \times 2}{\cancel{3}} = 40 \text{ weeks. } \textit{Ans.}$$

(Ex. 26.)—Tea is bought at $2/1\frac{1}{2}$ per lb. and sold at $2/6$ per lb.; how many pounds must be sold to gain £5?

Gain on each pound $= 2/6 - 2/1\frac{1}{2} = 4\frac{1}{2}$d. $£5 = 1200$d.

$$\begin{array}{lrl} & \text{Pence.} & \text{lbs.} \\ \text{No. of pounds to gain} & 4\frac{1}{2} & = 1 \\ \quad ,, \qquad\quad ,, & 1 & = \dfrac{1}{4\frac{1}{2}} \\ \quad ,, \qquad\quad ,, & 1200 & = \dfrac{1 \times 1200}{4\frac{1}{2}} \end{array}$$

$$\frac{1 \times 1200}{4\frac{1}{2}} = \frac{\frac{1}{1} \times \frac{1200}{1}}{\frac{9}{2}} = \frac{\overset{400}{\cancel{1200}} \times 2}{\underset{3}{\cancel{9}}} = \frac{400 \times 2}{3} = \frac{800}{3} = 266\tfrac{2}{3}\text{lbs. } \textit{Ans.}$$

(Ex. 27.)—If $3\frac{1}{4}$lbs. of tea cost $10/6\frac{3}{4}$, how much will $7\frac{1}{2}$lbs. cost?

$$\begin{array}{lrl} & \text{lbs.} & \\ \text{Cost of} & 3\frac{1}{4} & = 10/6\frac{3}{4} \\ \quad ,, & 1 & = \dfrac{10/6\frac{3}{4}}{3\frac{1}{4}} \\ \quad ,, & 7\frac{1}{2} & = \dfrac{10/6\frac{3}{4} \times 7\frac{1}{2}}{3\frac{1}{4}} \end{array}$$

$$\frac{10/6\frac{3}{4} \times 7\frac{1}{2}}{3\frac{1}{4}} = \frac{\frac{10/6\frac{3}{4}}{1} \times \frac{15}{2}}{\frac{13}{4}} = \frac{10/6\frac{3}{4} \times 15 \times \overset{2}{\cancel{4}}}{13 \times \cancel{2}} = \frac{£15\ 16\text{s. } 10\frac{1}{2}\text{d.}}{13}$$

$$= £1\ 4\text{s. } 4\tfrac{1}{2}\text{d. } \textit{Ans.}$$

Or, $3\frac{1}{4}$lbs. = 13 qr. lbs.; $7\frac{1}{2}$lbs. = 30 qr. lbs.

$$\begin{array}{rl} & \text{Qr. lbs.} \\ \text{Cost of} & 13 = 10/6\tfrac{3}{4} \\ \text{,,} & 1 = \dfrac{10/6\frac{3}{4}}{13} \\ \text{,,} & 30 = \dfrac{10/6\frac{3}{4} \times 30}{13} \end{array}$$

$$\frac{10/6\frac{3}{4} \times 30}{13} = \frac{\text{£}15\ 16\text{s.}\ 10\frac{1}{2}\text{d.}}{13} = \text{£}1\ 4\text{s.}\ 4\tfrac{1}{2}\text{d.} \quad \textit{Ans.}$$

(Ex. 28.)—How many lbs. of coffee at 1/6 per lb. are equal in value to 3lbs. of tea at 3/- per lb.?

3/- = 36d. 1/6 = 18d.

Consider the coffee at 1/6 as tea of that price: the question then becomes—How many lbs. of tea at 18d. per lb. are equal in value to 3lbs. at 36d. per lb.?

$$\begin{array}{lllll} & & \text{Pence.} & & \text{Pounds.} \\ \text{No. of pounds at} & & 36 & \text{per lb.} & = 3 \\ \text{,,} & \text{,,} & 1 & \text{,,} & = 3 \times 36 \\ \text{,,} & \text{,,} & 18 & \text{,,} & = \dfrac{3 \times 36}{18} \end{array}$$

$$\frac{3 \times \overset{2}{\cancel{36}}}{\cancel{18}} = 3 \times 2 = 6\text{lb.} \quad \textit{Ans.}$$

(Ex. 29.)—A postman walks 14 miles a day and takes three steps in every two yards; how many steps does he take in a working week?

No. of yards walked in a working week =

$1760 \times 14 \times 6 = 147840$yds.

$$\begin{array}{lllr} & & & \text{Yds. Steps} \\ \text{No. of steps in walking} & & & 2 = 3 \\ \text{,,} & \text{,,} & \text{,,} & 1 = \dfrac{3}{2} \\ \text{,,} & \text{,,} & & 147840 = \dfrac{3 \times 147840}{2} \end{array}$$

$$\frac{3 \times \overset{73920}{\cancel{147840}}}{\cancel{2}} = 221760 \text{ steps.} \quad \textit{Ans.}$$

(Ex. 30.)—I borrowed of my friend £64 for 8 months; he wants to borrow a sum of money of me for 12 months; how much must I lend him?

$$\begin{array}{lll} & \text{Months.} & \pounds \\ \text{Amount borrowed for} & 8 & =64 \\ \quad ,, \quad\quad ,, & 1 & =64\times 8 \\ \quad ,, \quad\quad ,, & 12 & =\dfrac{64\times 8}{12} \end{array}$$

$$\frac{\pounds 64\times \overset{2}{\cancel{8}}}{\underset{3}{\cancel{12}}}=\frac{\pounds 64\times 2}{3}=\frac{\pounds 128}{3}=\pounds 42\ 13s.\ 4d.\quad \textit{Ans.}$$

The reasoning in the unity line is similar to that in Ex. 17.

(Ex. 31.)—If 9 men can do a piece of work in 14 days, working 10 hours a day, how many hours a day must 12 men work to do the same work in 10 days?

This, though really a Compound Proportion sum, may be worked as a Simple Proportion, in *two* operations.

Number of hours for the 9 men $=14\times 10=140$ hours.

$$\begin{array}{lll} & \text{Men.} & \text{Hours.} \\ \text{No of hours for} & 9 & =140 \\ \quad ,, & 1 & =140\times 9 \\ \quad ,, & 12 & =\dfrac{140\times 9}{12} \end{array}$$

$$\frac{\overset{35}{\cancel{140}}\times \overset{3}{\cancel{9}}}{\underset{\cancel{4}}{\cancel{12}}}=105 \text{ hours.}$$

But these 105 hours belong to 10 working days—

$\therefore\ 105\div 10=10\frac{1}{2}$ hours. *Ans.*

(Ex. 32.)—Eight men do a piece of work in 6 days; in what time will 12 men do twice as much?

Two operations will be required in this sum.

$$\begin{array}{rl} & \text{Men. Days.} \\ \text{Time for } & 8 = 6 \\ ,, & 1 = 6 \times 8 \\ ,, & 12 = \dfrac{6 \times 8}{12} \end{array}$$

$$\frac{\cancel{6} \times \overset{4}{\cancel{8}}}{\underset{2}{\cancel{12}}} = 4 \text{ days.}$$

But 4 days is the time for an *equal* piece of work, and the question asks for twice as much work, therefore it will take twice as long,

$\therefore$ 4 days $\times$ 2 = 8 days. *Ans.*

24. Double Rule of Three or Compound Proportion.—The problems included under this rule are of a somewhat more complex nature than those just illustrated. Reduced to their simplest form, however, they consist, like Simple Rule of Three, of two parts.

(*a*) *The statement*—consisting of *more than two* terms, *all* of which are given.

(*b*) *The demand*—containing the same number of terms as the statement, all of which are given except *one*, which has to be found.

25. As many steps are required as there are terms in the question. The several terms of the statement are, one by one, reduced to unity; the given terms in the demand are then introduced, one by one, until a compound fraction is formed containing all the terms. This fraction, when reduced, gives the demand.

26. The following rules must be carefully observed (see pars. **12**, p. 13, and **20**, p. 17) :—

(*a*) The *statement* must be as concise as possible, only two or three words being used.

(*b*) It must be so arranged that the last of the terms in it is of the same kind as the required answer.

(*c*) As far as possible all the wording must be placed *before the first term* of the first step, and no grammatical alteration (except very rarely) is afterwards to be made in it.

27. The following example, reasoned out, shows the various steps of the method, and a similar plan will resolve all questions of a like character, however many terms they may contain.

(Ex. 33.)—If the wages of 6 men for 3 weeks be £27, what will be the wages of 4 men for 5 weeks?

The two parts of this problem are*—

(*a*) *statement*—the wages earned by 6 men in 3 weeks is £27.

(*b*) *demand*—the wages earned by 4 men in 5 weeks.

Statement : *1st step.*—Wages earned by 6 men in 3 weeks = £27.

Reducing the men to unity; since 1 man will earn 6 times less than 6 men, we get—

2nd step.—Wages earned by 1 man in 3 weeks $= \frac{£27}{6}$

Leaving this fraction to be afterwards resolved, and reducing the *weeks* to unity; since 3 times less will be earned in 1 week than in 3 weeks, we get—

3rd step.—Wages earned by 1 man in 1 week $= \frac{£27}{6 \times 3}$

Introducing the first term, *men*, of the demand; since 4 men in 1 week will earn 4 times more than 1 man in 1 week, we get—

4th step.—Wages earned by 4 men in 1 week $= \frac{£27 \times 4}{6 \times 3}$

---

* In a Notes of Lesson all the steps should be reasoned out, as in Ex. 33.

Introducing the second term, *weeks*, of the demand; since 4 men in 5 weeks will earn 5 times more than 4 men in 1 week, we get—

5th step.—Wages earned by 4 men in 5 weeks $=\frac{£27 \times 4 \times 5}{6 \times 3}$

Reducing the complex fraction we get—

6th step $\frac{£\overset{\overset{3}{\cancel{9}}}{\cancel{27}} \times \overset{2}{\cancel{4}} \times 5}{\underset{\cancel{2}}{\cancel{6}} \times \cancel{3}} = £3 \times 2 \times 5 = £30.$ *Ans.*

Without the train of reasoning the work appears thus on the child's slate or paper (excepting, of course, the numbering of the steps).

			Men.		Weeks.	£
1st step.—Wages	earned	by	6	in	3	$=27$
2nd step.	"	"	1	"	3	$=\frac{27}{6}$
3rd step, or Unity line.	"	"	1	"	1	$=\frac{27}{6 \times 3}$
4th step.	"	"	4	"	1	$=\frac{27 \times 4}{6 \times 3}$
5th step.	"	"	4	"	5	$=\frac{27 \times 4 \times 5}{6 \times 3}$

6th step. $\frac{£\overset{\overset{3}{\cancel{9}}}{\cancel{27}} \times \overset{2}{\cancel{4}} \times 5}{\underset{\cancel{2}}{\cancel{6}} \times \cancel{3}} = £3 \times 2 \times 5 = £30.$ *Ans.*

28. Arrangement of Work in Compound Proportion.—The above working, though both clear and simple, is, in school, seldom done neatly and tidily. The large number of figures at the ends of the lines are a cause of great trouble to children and of annoyance to teachers, and form one of the chief objections to this method of working Double Rule of Three. It therefore becomes necessary to resort to an arrangement which requires the formation of *one fraction only*, whilst all the

steps of the method are plainly indicated. This manner of arrangement, now to be explained, has stood the test of many years' work in school with unvarying success.

29. In actual working proceed thus :—

(1) Make the complete statement, as in the first step of the previous example.

(2) Reduce all the terms before the equality sign (=) to unity, *but without forming any of the fractions expressing their value.*

(3) Introduce, step by step, all the terms of the demand, but without forming the fractions expressing their value.

(4) Join all the steps, from the *second* to the *last*, with a brace ($\}$) on the right-hand side of the terms before the equality sign.

Exercise 33 will therefore appear, up to this point, as below :—

		Men.		Weeks.	£
1st step.—Wages	earned	by 6	in	3	= 27
"	"	1	"	3	
"	"	1	"	1	
"	"	4	"	1	
"	"	4	"	5	

Now proceed to form the fraction.

(1) Place an equality sign (=) at the point of the brace; in a line with it draw the line separating the numerator and denominator of the fraction now to be formed, and place the £27 as the first term in the numerator of this fraction, thus—

$$\Big\} = \frac{27}{\qquad\qquad}$$

(2) Take each step, beginning with the second, and, *according to the reasoning*, insert the term which is reduced to unity, either as numerator or denominator of the fraction, thus—

2nd step.—Insert the 6 men $= \dfrac{27}{6}$

$\left.\begin{array}{l}\textit{3rd step,}\\ \textit{or Unity}\\ \textit{line}\end{array}\right\}$ Next the 3 weeks $= \dfrac{27}{6 \times 3}$ $\Big\{$ All the terms of the demand are now reduced to unity.

4th step—Next the 4 men $= \dfrac{27 \times 4}{6 \times 3}$

5th step—Next the 5 weeks $= \dfrac{27 \times 4 \times 5}{6 \times 3}$

(3) Work out the fraction as before. The complete working will therefore appear as below—*

$$\begin{array}{lllcl} & & \text{Men.} & \text{Weeks.} & \pounds \\ \text{Wages} & \text{earned} & \text{by } 6 & \text{in } 3 & = 27 \\ \text{"} & \text{"} & 1 & \text{" } 3 & \multirow{4}{*}{$\left.\begin{array}{l} \\ \\ \\ \\ \end{array}\right\} = \dfrac{27 \times 4 \times 5}{6 \times 3}$} \\ \text{"} & \text{"} & 1 & \text{" } 1 & \\ \text{"} & \text{"} & 4 & \text{" } 1 & \\ \text{"} & \text{"} & 4 & \text{" } 5 & \end{array}$$

$$\frac{\pounds\overset{\overset{3}{\cancel{9}}}{\cancel{27}} \times \overset{2}{\cancel{4}} \times 5}{\underset{\cancel{2}}{\cancel{6}} \times \cancel{3}} = \pounds 3 \times 2 \times 5 \times \pounds 30. \quad \textit{Ans.}$$

(Ex. 34.)—If 13 men earn £30 in 14 days, how much will 16 men earn in 10 days at the same rate?

The first stage of the work will be as follows:—

$$\begin{array}{lllcl} & & \text{Men.} & \text{Days.} & \pounds \\ \text{Wages} & \text{earned} & \text{by } 13 & \text{in } 14 & = 30 \\ \text{"} & \text{"} & 1 & \text{" } 14 & \\ \text{"} & \text{"} & 1 & \text{" } 1 & \\ \text{"} & \text{"} & 16 & \text{" } 1 & \\ \text{"} & \text{"} & 16 & \text{" } 10 & \end{array}\left.\right\}$$

* In Appendix I., p. 108, the sums are worked alternately by this method and by the plan adopted on p. 34. Some teachers prefer the longer method to this shorter one.

Forming the fraction, as the reasoning indicates, and reducing, the working appears thus—

$$\begin{array}{llcl} & \text{Men.} & \text{Days.} & \text{£} \\ \text{Wages earned by} & 13 \text{ in} & 14 & =30 \\ \quad " \qquad\quad " & 1\ " & 14 & \\ \quad " \qquad\quad " & 1\ " & 1 & \\ \quad " \qquad\quad " & 16\ " & 1 & \\ \quad " \qquad\quad " & 16\ " & 10 & \end{array}\left.\vphantom{\begin{array}{l}1\\1\\1\\1\end{array}}\right\} = \frac{30 \times 16 \times 10}{13 \times 14}$$

$$\frac{\text{£}30 \times 16 \times \overset{5}{\cancel{10}}}{13 \times \underset{7}{\cancel{14}}} = \frac{\text{£}30 \times 80}{91} = \frac{\text{£}2400}{91} = \text{£}26\ 7\text{s. } 5\tfrac{1}{2}\text{d.} + \tfrac{62}{91}. \quad \textit{Ans.}$$

(Ex. 35.)—If 4 horses eat 3 bushels of oats in 9 days, how many bushels will 17 horses eat in 10 days?

$$\begin{array}{llcl} & \text{Horses.} & \text{Days.} & \text{Bushels.} \\ \text{Bushels eaten by} & 4 \text{ in} & 9 & =3 \\ \quad " \qquad\quad " & 1\ " & 9 & \\ \quad " \qquad\quad " & 1\ " & 1 & \\ \quad " \qquad\quad " & 17\ " & 1 & \\ \quad " \qquad\quad " & 17\ " & 10 & \end{array}\left.\vphantom{\begin{array}{l}1\\1\\1\\1\end{array}}\right\} = \frac{3 \times 17 \times 10}{4 \times 9}$$

$$\frac{\cancel{3} \times 17 \times \overset{5}{\cancel{10}}}{\underset{2}{\cancel{4}} \times \underset{3}{\cancel{9}}} = \frac{85}{6} = 14\tfrac{1}{6} \text{ bushels.} \quad \textit{Ans.}$$

(Ex. 36.)—If the carriage of 1 ton for 8 miles be £3 16s., what will be the charge for 4 tons for 12 miles?

$$\begin{array}{llcl} & \text{Tons.} & \text{Miles.} & \\ \text{Carriage of} & 1 \text{ for} & 8 & =\text{£}3\ 16\text{s.} \\ \qquad " & 1\ " & 1 & \\ \qquad " & 4\ " & 1 & \\ \qquad " & 4\ " & 12 & \end{array}\left.\vphantom{\begin{array}{l}1\\1\\1\end{array}}\right\} = \frac{\text{£}3\ 16\text{s.} \times 4 \times 12}{8}$$

$$\frac{\text{£}3\ 16\text{s.} \times \cancel{4} \times \overset{6}{\cancel{12}}}{\underset{\cancel{2}}{\cancel{8}}} = \text{£}3\ 16\text{s.} \times 6 = \text{£}22\ 16\text{s.} \quad \textit{Ans.}$$

The reasoning in the second and third steps requires careful attention.

(Ex. 37.)—Seven men can build a wall in 20 days, working 8 hours daily; how long will 4 men be in building it, working 10 hours daily?

The reasoning in the second and third steps requires careful attention.

	Men.		Hours.	Days.
Time for	7	working	8	=20
"	1	"	8	
"	1	"	1	$=\frac{20\times7\times8}{4\times10}$
"	4	"	1	
"	4	"	10	

$$\frac{\overset{2}{\cancel{20}}\times7\times\overset{2}{\cancel{8}}}{\cancel{4}\times\cancel{10}}=2\times7\times2=28 \text{ days. } \textit{Ans.}$$

(Ex. 38.)—If the wages for 6 men for 6 days be £9, what will be the wages of 15 men for 21 days?

	Men.		Days.	£
Wages for	6	for	6	=9
"	1	"	6	
"	1	"	1	$=\frac{9\times15\times21}{6\times6}$
"	15	"	1	
"	15	"	21	

$$£\frac{\overset{3}{\cancel{9}}\times15\times\overset{7}{\cancel{21}}}{\underset{2}{\cancel{6}}\times\underset{2}{\cancel{6}}}=£\frac{3\times15\times7}{2\times2}=£\frac{315}{4}=£78\ 15s. \quad \textit{Ans.}$$

(Ex. 39.)—If 12 horses plough 888 acres in 6 days, how many acres will 20 horses plough in 2 days?

		Horses.		Days.	Acres.
Acres ploughed	by	12	in	6	=888
"	"	1	"	6	
"	"	1	"	1	$=\frac{888\times20\times2}{12\times6}$
"	"	20	"	1	
"	"	20	"	2	

$$\frac{\overset{74}{\cancel{888}}\times20\times\cancel{2}}{\cancel{12}\times\underset{3}{\cancel{6}}}=\frac{74\times20}{3}=\frac{1480}{3}=493\tfrac{1}{3} \text{ acres. } \textit{Ans.}$$

(Ex. 40.)—What will it cost to keep 3 horses for half a year, if 4 horses for 3 months cost £30 12s. 6d.?

	Horses.		Months.		
Cost of	4	for	3		$= £30\ 12s.\ 6d.$
,,	1	,,	3		
,,	1	,,	1		$= \dfrac{£30\ 12s.\ 6d. \times 3 \times 6}{4 \times 3}$
,,	3	,,	1		
,,	3	,,	6		

$$\frac{£30\ 12s.\ 6d. \times 3 \times \overset{2}{\cancel{6}}}{\underset{2}{\cancel{4}} \times \cancel{3}} = \frac{£30\ 12s.\ 6d. \times 3}{2} = \frac{£91\ 17s.\ 6d.}{2}$$

$$= £45\ 18s.\ 9d. \quad \textit{Ans.}$$

(Ex. 41.)—What ought the sixpenny loaf to weigh, when wheat is 60/- a quarter, if the eightpenny loaf weighs 3lbs. when wheat is 54/- a quarter?

	Penny.			Shillings.	lbs.
Weight of	8	loaf with	wheat at	54	$= 3$
,,	1	,,	,,	54	
,,	1	,,	,,	1	$= \dfrac{3 \times 54 \times 6}{8 \times 60}$
,,	6	,,	,,	1	
,,	6	,,	,,	60	

$$\frac{\cancel{3} \times \overset{27}{\cancel{54}} \times \overset{3}{\cancel{6}}}{\underset{\underset{2}{\cancel{4}}}{\cancel{8}} \times \underset{20}{\cancel{60}}} = \frac{27 \times 3}{2 \times 20} = \frac{81}{40} = 2\tfrac{1}{40}\text{lbs.} \quad \textit{Ans.}$$

This sum (and the following one also) should be carefully noted. Incorrect reasoning in the third step almost invariably leads children (and often pupil-teachers) to place the 54 in the denominator, and, as a consequence, the 60 becomes a factor in the numerator. It is manifest, however, that if wheat is only 1s. a quarter, the baker can afford to let you have a larger loaf than when it is at 54s. per quarter. The statement, too, is an awkward one to make. It might be written thus :—

	Shillings.		Penny.		lbs.
With wheat at	54	weight of	8	loaf	$= 3$

(Ex. 42.)—If 400 soldiers eat 4 barrels of flour in 8 days, in what time will 600 soldiers eat 9 barrels?

$$\begin{array}{lccc} & \text{Soldiers.} & \text{Barrels.} & \text{Days.} \\ \text{Time for} & 400 \text{ to eat} & 4 & =8 \\ \quad " & 1 \quad " & 4 & \multirow{4}{*}{} \\ \quad " & 1 \quad " & 1 & \\ \quad " & 600 \quad " & 1 & \\ \quad " & 600 \quad " & 9 & \end{array}\left.\vphantom{\begin{array}{c}1\\1\\1\\1\end{array}}\right\} = \frac{8\times400\times9}{4\times600}$$

$$\frac{\overset{2}{\cancel{8}}\times\overset{2}{\cancel{400}}\times\overset{3}{\cancel{9}}}{\cancel{4}\times\underset{200}{\cancel{600}}}=2\times2\times3=12 \text{ days. } \textit{Ans.}$$

(Ex. 43.)—If 3 men can reap 8 acres of wheat in 2 days, how long will it take 5 men to reap 20 acres at the same rate?

$$\begin{array}{lccc} & \text{Men.} & \text{Acres.} & \text{Days.} \\ \text{Time for} & 3 \text{ to reap} & 8 & =2 \\ \quad " & 1 \quad " & 8 & \\ \quad " & 1 \quad " & 1 & \\ \quad " & 5 \quad " & 1 & \\ \quad " & 5 \quad " & 20 & \end{array}\left.\vphantom{\begin{array}{c}1\\1\\1\\1\end{array}}\right\} = \frac{2\times3\times20}{8\times5}$$

$$\frac{\cancel{2}\times3\times\overset{4}{\cancel{20}}}{\underset{2}{\cancel{8}}\times\cancel{5}}=3 \text{ days. } \textit{Ans.}$$

(Ex. 44.)—If 8 men reap a field of 6 acres in 3 days, in how many days will 6 men reap a field of 9 acres?

$$\begin{array}{lccc} & \text{Men.} & \text{Acres.} & \text{Days.} \\ \text{Time for} & 8 \text{ to reap} & 6 & =3 \\ \quad " & 1 \quad " & 6 & \\ \quad " & 1 \quad " & 1 & \\ \quad " & 6 \quad " & 1 & \\ \quad " & 6 \quad " & 9 & \end{array}\left.\vphantom{\begin{array}{c}1\\1\\1\\1\end{array}}\right\} = \frac{3\times8\times9}{6\times6}$$

$$\frac{\cancel{3}\times\overset{\overset{2}{\cancel{4}}}{\cancel{8}}\times\overset{3}{\cancel{9}}}{\underset{2}{\cancel{6}}\times\underset{2}{\cancel{6}}}=6 \text{ days. } \textit{Ans.}$$

(Ex. 45.)—If 20cwt. be carried 50 miles for £5, what will be paid for the carriage of 40cwt. for 100 miles?

$$
\begin{array}{lcclcl}
 & \text{Cwt.} & & \text{Miles.} & & \pounds \\
\text{Cost of} & 20 & \text{for} & 50 & = & 5 \\
\text{"} & 1 & \text{"} & 50 & \multirow{4}{*}{$\left.\begin{array}{c}\\ \\ \\ \\ \end{array}\right\}$} & \\
\text{"} & 1 & \text{"} & 1 & & =\dfrac{5\times 40\times 100}{20\times 50} \\
\text{"} & 40 & \text{"} & 1 & & \\
\text{"} & 40 & \text{"} & 100 & &
\end{array}
$$

$$\frac{\pounds 5\times \overset{2}{\cancel{40}}\times \overset{2}{\cancel{100}}}{\cancel{20}\times\cancel{50}}=\pounds 5\times 2\times 2=\pounds 20. \quad \textit{Ans.}$$

(Ex. 46.)—If 800 soldiers consume 5 sacks of flour in 6 days, how many will consume 15 sacks in 2 days?

$$
\begin{array}{lclcl}
 & \text{Sacks.} & & \text{Days.} & \text{Soldiers.} \\
\text{Soldiers to eat} & 5 & \text{in} & 6 & =800 \\
\text{"}\quad\text{"} & 1 & \text{"} & 6 & \\
\text{"}\quad\text{"} & 1 & \text{"} & 1 & =\dfrac{800\times 6\times 15}{5\times 2} \\
\text{"}\quad\text{"} & 15 & \text{"} & 1 & \\
\text{"}\quad\text{"} & 15 & \text{"} & 2 &
\end{array}
$$

$$\frac{800\times \overset{3}{\cancel{6}}\times \overset{3}{\cancel{15}}}{\cancel{5}\times\cancel{2}}=800\times 3\times 3=7200 \text{ soldiers.} \quad \textit{Ans.}$$

Notice the reasoning in the second and third steps.

(Ex. 47.)—If £60 support 8 persons for 4 months, how long ought £15 to maintain 6 persons at the same rate?

$$
\begin{array}{lclcl}
 & \pounds & & \text{Persons.} & \text{Months.} \\
\text{Time for} & 60 & \text{to support} & 8 & =4 \\
\text{"} & 1 & \text{"} & 8 & \\
\text{"} & 1 & \text{"} & 1 & =\dfrac{4\times 8\times 15}{60\times 6} \\
\text{"} & 15 & \text{"} & 1 & \\
\text{"} & 15 & \text{"} & 6 &
\end{array}
$$

$$\frac{\overset{4}{\cancel{4}}\times\cancel{8}\times\cancel{15}}{\underset{15}{\cancel{60}}\times\underset{3}{\cancel{6}}}=\frac{4}{3}\text{ month}=1\tfrac{1}{3}\text{ month.} \quad \textit{Ans.}$$

(Ex. 48.)—If 8 horses can be kept for 6 weeks for £12, what sum of money ought to keep 16 horses for 2 weeks?

$$\begin{array}{l} \quad\text{Horses. Weeks.}\ \ \pounds \\ \text{Cost of } 8 \text{ for } 6 \ = 12 \\ \left.\begin{array}{lrlr} \text{,,} & 1 & \text{,,} & 6 \\ \text{,,} & 1 & \text{,,} & 1 \\ \text{,,} & 16 & \text{,,} & 1 \\ \text{,,} & 16 & \text{,,} & 2 \end{array}\right\} = \frac{12 \times 16 \times 2}{8 \times 6} \end{array}$$

$$\frac{\pounds \overset{2}{\cancel{12}} \times \overset{2}{\cancel{16}} \times 2}{\cancel{8} \times \cancel{6}} = \pounds 2 \times 2 \times 2 = \pounds 8. \quad \textit{Ans.}$$

(Ex. 49.)—If 16 horses eat 96 bushels of corn in 42 days, in how many days will 7 horses eat half as much?

Half as much $= 96 \div 2 = 48$ bushels.

$$\begin{array}{l} \qquad\quad\text{Horses. Bushels. Days.} \\ \text{Days for } 16 \text{ to eat } 96 \ = 42 \\ \left.\begin{array}{lrlr} \text{,,} & 1 & \text{,,} & 96 \\ \text{,,} & 1 & \text{,,} & 1 \\ \text{,,} & 7 & \text{,,} & 1 \\ \text{,,} & 7 & \text{,,} & 48 \end{array}\right\} = \frac{42 \times 16 \times 48}{96 \times 7} \end{array}$$

$$\frac{\overset{6}{\cancel{42}} \times \cancel{16} \times 48}{\underset{6}{\cancel{96}} \times \cancel{7}} = 48 \text{ days.} \quad \textit{Ans.}$$

The second term in the fifth step might have been stated thus—$\frac{96}{2}$, and the fraction formed as below:—

$$\text{Days for 7 to eat } \frac{96}{2} = \frac{\overset{6}{\cancel{42}} \times \cancel{16} \times \overset{8}{\cancel{96}}}{\cancel{96} \times \cancel{7} \times \cancel{2}} = 48 \text{ days.} \quad \textit{Ans.}$$

(Horses. Bushels.)

(Ex. 50.)—If 12 horses in 5 days draw 44 tons of stone, how many horses will draw 8 times as much, the same distance, in 18 days?

Eight times as much $= 44 \times 8$.

$$\begin{array}{l} \qquad\qquad\qquad\text{Tons. Days. Horses.} \\ \left.\begin{array}{r}\text{No. of horses}\\ \text{to draw}\end{array}\right\} 44 \text{ in } 5 \ = 12 \\ \left.\begin{array}{lrlr} \text{,,} & 1 & \text{,,} & 5 \\ \text{,,} & 1 & \text{,,} & 1 \\ \text{,,} & (44 \times 8) & \text{,,} & 1 \\ \text{,,} & (44 \times 8) & \text{,,} & 18 \end{array}\right\} = \frac{12 \times 5 \times 44 \times 8}{44 \times 18} \end{array}$$

$$\frac{\overset{2}{\cancel{12}} \times 5 \times \cancel{44} \times 8}{\cancel{44} \times \underset{3}{\cancel{18}}} = \frac{2 \times 5 \times 8}{3} = \frac{80}{3} = 26\tfrac{2}{3} \text{ horses.} \quad \textit{Ans.}$$

Notice the new term in the fourth step. There is no need to multiply the 44 and 8 together. The answer should properly be 27 *horses*, as $\frac{2}{3}$ of a horse is an impossible thing. Children are often puzzled with answers like this.

Where space is limited in width, the above way of arranging the statement line is very convenient.

(Ex. 51.)—If 6 bars of iron 4ft. long, 3in. broad, and 2in. thick, weigh 288lbs., how much will 15 weigh, each 6ft. long, 4in. broad, and 3in. thick?

	Bars.		Feet.		Inches.		Inches.		lbs.
Weight of	6	each	4	long,	3	broad,	2	thick	= 288
„	1	„	4	„	3	„	2	„	
„	1	„	1	„	3	„	2	„	
„	1	„	1	„	1	„	2	„	
„	1	„	1	„	1	„	1	„	
„	15	„	1	„	1	„	1	„	
„	15	„	6	„	1	„	1	„	
„	15	„	6	„	4	„	1	„	
„	15	„	6	„	4	„	3	„	

$$= \frac{288\times15\times6\times4\times3}{6\times4\times3\times2}$$

$$\frac{\overset{144}{\cancel{288}}\times15\times\cancel{6}\times\cancel{4}\times\cancel{3}}{\cancel{6}\times\cancel{4}\times\cancel{3}\times\cancel{2}}=144\times15=2160\text{lbs.}=19\text{cwt. }1\text{qr. }4\text{lbs.}\quad \textit{Ans.}$$

Or, since the weights will be in proportion to the *cubical contents* of each bar, the steps may be shortened, and the question worked as a simple Rule of Three, thus :—

(4ft. × 12in. × 3in. × 2in.) × 6 = cubical contents, in inches, of 6 bars.
(6ft. × 12in. × 4in. × 3in.) × 15 = „ „ „ 15 „

	Cubic inches.	lbs.
Weight of	$(4\times12\times3\times2\times6)$	$=288$
„	1	$=\dfrac{288}{4\times12\times3\times2\times6}$
„	$(6\times12\times4\times3\times15)$	$=\dfrac{288\times6\times12\times4\times3\times15}{4\times12\times3\times2\times6}$

$$\frac{\overset{144}{\cancel{288}}\times\cancel{6}\times\cancel{12}\times\cancel{4}\times\cancel{3}\times15}{\cancel{4}\times\cancel{12}\times\cancel{3}\times\cancel{2}\times\cancel{6}}=2160\text{lbs.}=19\text{cwt. }1\text{qr. }4\text{lbs.}\quad \textit{Ans.}$$

30. Problems Involving Vulgar Fractions.—If the children have had a fair amount of practice in the reducing of compound and complex fractions (see par. **15**, p. 15; and par. **22**, p. 26), they will have little difficulty in working problems such as the following:—

(Ex. 52.)—If $\frac{2}{9}$ of a ton of coals cost 4/-, what is the value of $\frac{3}{4}$ of a ton?

$$\begin{array}{lll} & \text{Tons.} & \text{Shillings.} \\ \text{Cost of} & \frac{2}{9} & = 4 \\ \quad " & \frac{1}{9} & = \frac{4}{2} \\ \therefore \quad " & \frac{9}{9} \text{ (or 1 ton)} & = \frac{4 \times 9}{2} = 2 \times 9 = 18/- \\ \text{And} \quad " & \frac{3}{4} & = \frac{18 \times 3}{4}. \end{array}$$

$$\frac{\overset{9}{\cancel{18}} \times 3}{\underset{2}{\cancel{4}}} = \frac{27}{2} = 13/6. \quad \textit{Ans.}$$

In practice the *second* step in the above working may be omitted, for it is evident that if

$$\frac{1}{9} = \frac{4}{2}, \text{ and } \frac{9}{9} = \frac{4 \times 9}{2},$$

the last fraction can be obtained at one operation, thus—

$$\begin{array}{ll} \text{Cost of } \frac{2}{9} \text{ tons} & = 4 \text{ shillings} \\ \quad " \quad \frac{9}{9} \quad " & = \frac{\frac{4}{2}}{9}, \end{array}$$

and this complex fraction is simply

$$\frac{\frac{4}{1}}{\frac{2}{9}} = \frac{4 \times 9}{2}$$

as before. The *second* and *third* steps are therefore reduced to one step only, and the working appears in this form :—

$$\begin{array}{lll}
\text{Tons. Shillings.} & & \\
\text{Cost of } \frac{2}{9}=4 & & \\
\text{,,} \quad 1=\dfrac{4}{\frac{2}{9}} & \left(\begin{array}{c}\text{Or, at teacher's}\\ \text{discretion}\end{array}\right) & =\dfrac{\frac{4}{1}}{\frac{2}{9}} \\
\text{,,} \quad \frac{3}{4}=\dfrac{4\times\frac{3}{4}}{\frac{2}{9}} & \text{(Ditto)} & =\dfrac{\frac{4}{1}\times\frac{3}{4}}{\frac{2}{9}}
\end{array}$$

$$\frac{4\times\frac{3}{4}}{\frac{2}{9}}=\frac{\frac{4}{1}\times\frac{3}{4}}{\frac{2}{9}}=\frac{\cancel{4}\times3\times9}{\cancel{4}\times2}=\frac{27}{2}=13/6. \quad \textit{Ans.}$$

(Ex. 53.)—If $15\frac{5}{8}$ yards cost $12\frac{1}{3}$ shillings, what will $4\frac{5}{6}$ yards cost?

$$\begin{array}{l}
\text{Yards. Shillings.} \\
\text{Cost of } 15\frac{5}{8}=12\frac{1}{3} \\
\text{,,} \quad 1 = \dfrac{12\frac{1}{3}}{15\frac{5}{8}} \\
\text{,,} \quad 4\frac{5}{6}=\dfrac{12\frac{1}{3}\times4\frac{5}{6}}{15\frac{5}{8}}
\end{array}$$

$$\frac{\frac{37}{3}\times\frac{29}{6}}{\frac{125}{8}}=\frac{37\times29\times\overset{4}{\cancel{8}}}{3\times\underset{3}{\cancel{6}}\times125}=\frac{4292}{1125}=3/9\tfrac{293}{375} \text{ or } 3/9\tfrac{3}{4}+\tfrac{47}{375}. \quad \textit{Ans.}$$

(Ex. 54.)—If $3\frac{3}{4}$lbs. of tea cost 10/4, what will $13\frac{7}{8}$lbs. cost at the same rate?

$$\begin{array}{l}
\text{lbs.} \\
\text{Cost of } 3\frac{3}{4}=10/4 \\
\text{,,} \quad 1 = \dfrac{10/4}{3\frac{3}{4}} \\
\text{,,} \quad 13\frac{7}{8}=\dfrac{10/4\times13\frac{7}{8}}{3\frac{3}{4}}
\end{array}$$

$$\frac{\frac{10/4}{1}\times\frac{111}{8}}{\frac{15}{4}}=\frac{\overset{5/2}{\cancel{10/4}}\times\overset{37}{\cancel{111}}\times\cancel{4}}{\underset{\underset{1}{\cancel{2}}}{\cancel{8}}\times\underset{5}{\cancel{15}}}=\frac{5/2\times37}{5}=\frac{£9\ 11s.\ 2d.}{5}=£1\ 18s.\ 2\tfrac{3}{4}d.+\tfrac{1}{5}.$$

Or = £1 18s. $2\frac{4}{5}$d. *Ans.*

(Ex. 55.)—My uncle left me $\frac{3}{4}$ of an estate. I sold $\frac{1}{6}$ of my share for £1250; what was the value of the estate?

$$\frac{1}{6} \text{ of } \frac{3}{4} \text{ of the estate} = \frac{1}{\cancel{6}_2} \times \frac{\cancel{3}}{4} = \frac{1}{8}$$

Estate. £

$$\text{Value of } \frac{1}{8} = 1250$$

$$\text{,,} \quad 1 = \frac{1250}{\frac{1}{8}}$$

$$\frac{\frac{£1250}{1}}{\frac{1}{8}} = £1250 \times 8 = £10{,}000. \quad \textit{Ans.}$$

(Ex. 56.)—What must be given for $3\frac{1}{2}$cwt. of coal, if $\frac{3}{7}$ of a ton cost 7/9?

$$\frac{3}{7} \text{ ton} = \frac{3}{7} \times \frac{20}{1} = \frac{60}{7} \text{cwt.} \qquad 7/9 = 7\tfrac{3}{4}\text{s.}$$

Cwt. Shil.

$$\text{Cost of } \frac{60}{7} = 7\tfrac{3}{4}$$

$$\text{,,} \quad 1 = \frac{7\frac{3}{4}}{\frac{60}{7}}$$

$$\text{,,} \quad 3\tfrac{1}{2} = \frac{7\frac{3}{4} \times 3\frac{1}{2}}{\frac{60}{7}}$$

$$\frac{\frac{31}{4} \times \frac{7}{2}}{\frac{60}{7}} = \frac{31 \times 7 \times 7}{4 \times 60 \times 2} = \frac{1519}{480} = 3/1\tfrac{3}{4} + \tfrac{9}{10}. \quad \textit{Ans.}$$

(Ex. 57.)—How much tea can I buy for £1 15s. $8\frac{3}{4}$d., when I pay 14/- for $4\frac{4}{5}$lbs.?

14/- = 168d. £1 15s. $8\frac{3}{4}$d. = $428\frac{3}{4}$d.

	Pence.		lbs.
Pounds of tea for	168	=	$4\frac{4}{5}$
”	1	=	$\frac{4\frac{4}{5}}{168}$
”	$428\frac{3}{4}$	=	$\frac{4\frac{4}{5} \times 428\frac{3}{4}}{168}$

$$\frac{\frac{24}{5} \times \frac{1715}{4}}{\frac{168}{1}} = \frac{\overset{6}{\cancel{24}} \times \overset{\overset{49}{\cancel{343}}}{\cancel{1715}}}{\cancel{5} \times \cancel{4} \times \underset{\underset{4}{\cancel{28}}}{\cancel{168}}} = \frac{49}{4} = 12\tfrac{1}{4}\text{lbs.} \quad \textit{Ans.}$$

(Ex. 58.)—If $\frac{3}{13}$ of a wall is built in 26 days by 8 men working 9 hours a day, how much will be done by 12 men and 5 boys in 8 days, working 10 hours a day, if two boys do as much as one man?

$$5 \text{ boys} = \frac{5}{2} = 2\tfrac{1}{2} \text{ men} \quad \therefore \; 12 \text{ men} + 2\tfrac{1}{2} \text{ men} = 14\tfrac{1}{2} \text{ men.}$$

	Men.		Hours.		Days.	
Length of wall built by...	8	working	9	per day in	26	$= \frac{3}{13}$
”	1	”	9	”	26	
”	1	”	1	”	1	
”	$14\frac{1}{2}$	”	1	”	1	$= \frac{\frac{3}{13} \times 14\frac{1}{2} \times 10 \times 8}{8 \times 9 \times 26}$
”	$14\frac{1}{2}$	”	10	”	1	
”	$14\frac{1}{2}$	”	10	”	8	

$$\frac{\frac{3}{13} \times \frac{29}{2} \times \frac{10}{1} \times \frac{8}{1}}{\frac{8}{1} \times \frac{9}{1} \times \frac{26}{1}} = \frac{\cancel{3} \times 29 \times \overset{5}{\cancel{10}} \times \cancel{8}}{13 \times \cancel{2} \times \cancel{8} \times \underset{3}{\cancel{9}} \times 26} = \frac{29 \times 5}{13 \times 3 \times 26} = \frac{145}{1014}. \quad \textit{Ans.}$$

(Ex. 59.)—If 8 men can dig a trench 100ft. long, 3ft. broad, and 4ft. 6in. deep in 9 days, how many will be required to dig a trench 80ft. long, 5ft. broad, and 2ft. deep in $5\frac{1}{3}$ days?

$$4\text{ft. }6\text{in.}=4\tfrac{1}{2}\text{ feet.}$$

$$\begin{array}{llllllllllll}
 & \text{Feet.} & & \text{Feet.} & & \text{Feet.} & & \text{Days.} & \text{Men.} \\
\text{Number of men to dig} & 100 & \text{long,} & 3 & \text{broad,} & 4\tfrac{1}{2} & \text{deep in} & 9 & =8 \\
\text{,,} & 1 & \text{,,} & 3 & \text{,,} & 4\tfrac{1}{2} & \text{,,} & 9 & \\
\text{,,} & 1 & \text{,,} & 1 & \text{,,} & 4\tfrac{1}{2} & \text{,,} & 9 & \\
\text{,,} & 1 & \text{,,} & 1 & \text{,,} & 1 & \text{,,} & 9 & \\
\text{,,} & 1 & \text{,,} & 1 & \text{,,} & 1 & \text{,,} & 1 & \\
\text{,,} & 80 & \text{,,} & 1 & \text{,,} & 1 & \text{,,} & 1 & =\dfrac{8\times80\times5\times2\times9}{100\times3\times4\frac{1}{2}\times5\frac{1}{3}} \\
\text{,,} & 80 & \text{,,} & 5 & \text{,,} & 1 & \text{,,} & 1 & \\
\text{,,} & 80 & \text{,,} & 5 & \text{,,} & 2 & \text{,,} & 1 & \\
\text{,,} & 80 & \text{,,} & 5 & \text{,,} & 2 & \text{,,} & 5\tfrac{1}{3} &
\end{array}$$

$$\frac{\frac{8}{1}\times\frac{80}{1}\times\frac{5}{1}\times\frac{2}{1}\times\frac{9}{1}}{\frac{100}{1}\times\frac{3}{1}\times\frac{9}{2}\times\frac{16}{3}}=\frac{\cancel{8}\times\overset{4}{\cancel{80}}\times\cancel{5}\times2\times\cancel{9}\times\cancel{2}\times\cancel{3}}{\underset{5}{\cancel{100}}\times\cancel{3}\times\cancel{9}\times\underset{2}{\cancel{16}}}=8\text{ men. } \textit{Ans.}$$

This problem might have been worked like Ex. 51, by stating the cubical contents dug by each set of men thus—

$$\begin{array}{llll}
 & \text{C. feet.} & \text{Days.} & \text{Men.} \\
\text{Men to dig} & (100\times3\times4\tfrac{1}{2}) & \text{in } 9 & =8 \\
\text{,,} & 1 & \text{,, } 9 & \\
\text{,,} & 1 & \text{,, } 1 & =\dfrac{8\times80\times5\times2\times9}{100\times3\times4\frac{1}{2}\times5\frac{1}{3}} \\
\text{,,} & (80\times5\times2) & \text{,, } 1 & \\
\text{,,} & \text{,,} & \text{,, } 5\tfrac{1}{3} &
\end{array}$$

31. Problems Involving Decimal Fractions.—Excepting in cases where recurring decimals enter into the given data, it will rarely be necessary to reduce decimal fractions to vulgar ones before proceeding with the formation of the fraction which represents the answer. The placing of the decimal point in multiplication and division requires to be thoroughly understood.

(Ex. 60.)—If 3·25lbs. of tea cost 9·75 shillings, what is the value of 4·45lbs.?

$$\begin{array}{rl}
 & \text{Lbs. Shillings.} \\
\text{Cost of } & 3\cdot25=9\cdot75 \\
\text{,,} & 1=\dfrac{9\cdot75}{3\cdot25} \\
\text{,,} & 4\cdot45=\dfrac{9\cdot75\times4\cdot45}{3\cdot25}
\end{array}$$

$$\frac{\overset{3}{\cancel{9\cdot75}}\times4\cdot45}{\cancel{3\cdot25}}=4\cdot45\times3\text{ shil.}=13\cdot35\text{ shil.}(=13\tfrac{7}{20}\text{ shil.})=13/4\tfrac{1}{5}.\ \textit{Ans.}$$

(Ex. 61.)—If 29·2lbs. of ginger cost £4·33, find the cost of 50·23lbs.?

$$\begin{array}{rl} & \text{Lbs.} \quad \text{£} \\ \text{Cost of} & 29{\cdot}2 = 4{\cdot}33 \\ ,, & 1 = \dfrac{4{\cdot}33}{29{\cdot}2} \\ ,, & 50{\cdot}23 = \dfrac{4{\cdot}33 \times 50{\cdot}23}{29{\cdot}2} \end{array}$$

$$\frac{£4{\cdot}33 \times 50{\cdot}23}{29{\cdot}2} = £\frac{217{\cdot}4959}{29{\cdot}2} = £7{\cdot}4484 + = £7 \text{ 8s. } 11\tfrac{1}{2}\text{d.} + {\cdot}464\ldots \textit{ Ans.}$$

(Ex. 62.)—If 2lbs. of sugar cost ·086 of 12/-, what is the value of ·0625cwt.?

$$\begin{array}{c} {\cdot}086 \text{ of } 12/- = {\cdot}086 \times 12/- = 1{\cdot}032\text{s.} \\ {\cdot}0625\text{cwt.} = {\cdot}0625 \times 112 = 7\text{lbs.} \end{array}$$

$$\begin{array}{rl} & \text{lbs.} \quad \text{shil.} \\ \text{Cost of} & 2 = 1{\cdot}032 \\ ,, & 1 = \dfrac{1{\cdot}032}{2} \\ ,, & 7 = \dfrac{1{\cdot}032 \times 7}{2} \end{array}$$

$$\frac{\overset{{\cdot}516}{\cancel{1{\cdot}032}} \times 7}{\cancel{2}} = {\cdot}516 \times 7 = 3{\cdot}612/- = 3/7{\cdot}344 = 3/7\tfrac{1}{4} + {\cdot}376. \quad \textit{Ans.}$$

(Ex. 63.)—If $17\frac{3}{5}$yds. lace cost £2·56, how many yards will $13\frac{7}{8}$ guineas buy?

By vulgar fractions—

$$£2{\cdot}56 = £2\tfrac{14}{25}. \quad 13\tfrac{7}{8}\text{guin.} = £14\tfrac{91}{160}.$$

$$\begin{array}{rl} & \text{£} \quad \text{yds.} \\ \text{Yards for} & 2\tfrac{14}{25} = 17\tfrac{3}{5} \\ ,, & 1 = \dfrac{17\frac{3}{5}}{2\frac{14}{25}} \\ ,, & 14\tfrac{91}{160} = \dfrac{17\frac{3}{5} \times 14\frac{91}{160}}{2\frac{14}{25}} \end{array}$$

$$\frac{\frac{88}{5} \times \frac{2331}{160}}{\frac{64}{25}} = \frac{\overset{11}{\cancel{88}} \times 2331 \times \overset{5}{\cancel{25}}}{\cancel{5} \times \underset{\underset{4}{\cancel{32}}}{\cancel{160}} \times 64} = \frac{25641}{256} = 100\tfrac{41}{256}\text{yds.}$$

$$= 100\text{yds. 0ft. } 5\tfrac{3}{4}\text{in.} + \quad \textit{Ans.}$$

Or by decimal fractions :—

$17\frac{3}{5}$yds. $=17\cdot6$yds. $13\frac{7}{8}$guin. $=$ £$14\cdot56875$.

$$\begin{array}{llcl} & & \text{£} & \text{Yds.} \\ \text{Yards for} & & 2\cdot56 & =17\cdot6 \\ \quad ,, & & 1 & =\dfrac{17\cdot6}{2\cdot56} \\ \quad ,, & & 14\cdot56875 & =\dfrac{17\cdot6\times14\cdot56875}{2\cdot56} \end{array}$$

$$\frac{\overset{\overset{1\cdot1}{\cancel{4\cdot4}}}{\cancel{17\cdot6}}\times14\cdot56875}{\underset{\underset{\cdot16}{\cancel{\cdot64}}}{\cancel{2\cdot56}}}=\frac{16\cdot025625}{\cdot16}=100\cdot16015625\text{yds.},$$

or 100yds. 0ft. $5\frac{3}{4}$in. + *Ans.*

32. Complicated Problems.—The following are examples of somewhat more difficult problems, requiring closer reasoning than those which precede them. They will be especially useful and interesting to Pupil Teachers as showing the varied application of the Method.

(Ex. 64.)—A house is assessed to the poor rate at $\frac{20}{27}$ of its annual rental. If an assessment of 1/6 in the £ amounts to £2 10s., what is the rent?

£2 1 0 0 $=600$d. $1/6=18$d.

Then $600\text{d}\div18\text{d.}=$ number of pounds the house is assessed at $=$ £$33\frac{1}{3}$.

$$\begin{array}{lccl} & & & \text{£} \\ \text{Value of} & \dfrac{20}{27} & \text{of rent} & =33\frac{1}{3} \\ \quad ,, & \dfrac{1}{27} & ,, & =\dfrac{33\frac{1}{3}}{20} \\ \quad ,, & \dfrac{27}{27} & ,, & =\dfrac{33\frac{1}{3}\times27}{20} \end{array}$$

$$\text{£}\frac{33\frac{1}{3}\times27}{20}=\text{£}\frac{100\times27}{20\times3}=\text{£}\frac{2700}{60}=\text{£}45.\quad \textit{Ans.}$$

(Ex. 65.)—The expense of carpeting a room was £11 5s., but if the breadth had been 3 feet less than it was the expense would only have been £9. Find the breadth of the room.

By the question a difference of 3 feet more in breadth causes an extra expense of £11 5s. − £9 = £2 5s.

£2 5s. = 45s. £11 5s. = 225s.

$$\begin{array}{lrcl} & \text{Shil.} & & \text{Feet.} \\ \text{No. of feet in breadth costing} & 45 & = & 3 \\ \text{,,} \qquad \text{,,} & 1 & = & \frac{3}{45} \\ \text{,,} \qquad \text{,,} & 225 & = & \frac{3 \times 225}{45} \end{array}$$

$$\frac{\cancel{3} \times 225}{\underset{15}{\cancel{45}}} = 15 \text{ feet broad. } \textit{Ans.}$$

(Ex. 66.)—An officer receives £17 6s. 6d. pay for one month, the Income-tax of 3d. in the £ for the previous quarter having been deducted from it; find his income.

One month's pay = $\frac{1}{12}$ income.

Income-tax for a year = $\frac{3}{240}$ of income.

Income-tax for 3 months = $\frac{1}{4}$ of $\frac{3}{240}$ of income = $\frac{1}{4} \times \frac{1}{80} = \frac{1}{320}$.

∴ $\frac{1}{12}$ income − $\frac{1}{320}$ = part of income left after deducting tax.

$= \frac{80-3}{960} = \frac{77}{960} =$,, ,, ,,

Value of $\frac{77}{960}$ of income = £17 6s. 6d.

,, $\frac{1}{960}$,, $= \frac{\text{£}17 \text{ 6s. 6d.}}{77}$

,, $\frac{960}{960}$,, $= \frac{\text{£}17 \text{ 6s. 6d.} \times 960}{77}$

$$\frac{\text{£}17 \text{ 6s. 6d.} \times 960}{77} = \frac{\text{£}16632}{77} = \text{£}216. \quad \textit{Ans.}$$

(Ex. 67.)—If Fanny can write 9 lines while Mary writes 14, and Mary can write 7 while Kate writes 9, how many ought Kate to write in the time that Fanny writes 30?

In order to see plainly what has to be done, first arrange the terms of the question thus—

Fanny.		Mary.		Kate.
9	=	14		
		7	=	9

(a) The question is first to find out how many Kate would write in the time that Fanny does 9 and Mary 14 lines.

$$\begin{array}{lll} \text{If} & 7 \text{ of Mary's} & = 9 \text{ of Kate's} \\ \text{Then } 1 & \quad ,, & = \frac{9}{7} \quad ,, \\ \text{And } 14 & \quad ,, & = \frac{9 \times 14}{7} \; ,, = 18 \text{ lines.} \end{array}$$

(b) The question now becomes, if 18 of Kate's lines = 9 of Fanny's, how many will equal 30 of Fanny's?

$$\begin{array}{lll} \text{If} & 9 \text{ of Fanny's} & = 18 \text{ of Kate's} \\ \text{Then } 1 & \quad ,, & = \frac{18}{9} \quad ,, \\ \text{And } 30 & \quad ,, & = \frac{18 \times 30}{9} \; ,, = 60 \text{ lines.} \quad \textit{Ans.} \end{array}$$

The above method of arranging the work is applicable to all sums of this class. Though rather long in working, the reasoning is easily understood by children, especially if the new terms are inserted thus.

Fanny.		Mary.		Kate.
9	=	14	=	[18 (a)]
		7	=	9
∴ 30			=	[60 (b)].. *Ans.*

(Ex. 68.)—Divide 56 into three such parts that when respectively divided by 3, 4 and 5, the quotients may be in the same proportion as 6, 7 and 8.

First multiply the divisors by their respective proportional numbers to get them proportionate.

$$\left.\begin{array}{l} 3 \times 6 = 18 \\ 4 \times 7 = 28 \\ 5 \times 8 = 40 \end{array}\right\} = 86 : \text{the parts into which 56 must be divided.}$$

(a) If the proportion for 86 parts = 18 parts

$$\begin{array}{llll} \text{then} & ,, & 1 \; ,, & = \frac{18}{86} \quad ,, \\ \text{and} & ,, & 56 \; ,, & = \frac{18 \times 56}{86} = 11\tfrac{31}{43}. \quad \textit{Ans.} \end{array}$$

(b) If the proportion for 86 parts = 28 parts,

$$\begin{array}{llll} \text{then} & ,, & 1 \; ,, & = \frac{28}{86} \quad ,, \\ \text{and} & ,, & 56 \; ,, & = \frac{28 \times 56}{86} = 18\tfrac{10}{43}. \quad \textit{Ans.} \end{array}$$

(c) If the proportion for 86 parts $=40$ parts,

then „ 1 „ $=\frac{40}{86}$ „

and „ 56 „ $=\frac{40\times 56}{86}=26\frac{2}{43}$. *Ans.*

Proof. Total of Answers a, b, and $c=\underline{\underline{56}}$

(Ex. 69.)—A trench 920 feet long, 17 feet wide, and 10 feet deep has been dug by 7 men and 2 boys. It could have been done in the same time by 6 men and 5 boys. What length of trench 15 feet wide, and 12 feet deep, could have been dug by 5 men and 3 boys in half the time?

First reduce the men to boys.

$$7 \text{ men} + 2 \text{ boys} = 6 \text{ men} + 5 \text{ boys}$$
$$\text{or } 7 \text{ men} - 6 \text{ men} = 5 \text{ boys} - 2 \text{ boys}$$
$$\text{i.e., } 1 \text{ man} = 3 \text{ boys}$$
$$\therefore\ 6 \text{ men} + 5 \text{ boys} = (6\times 3)+5 = 23 \text{ boys}$$
$$\text{and } 5 \text{ men} + 3 \text{ boys} = (5\times 3)+3 = 18 \text{ boys.}$$

			Boys.	Feet.
Length of trench	(17ft. by 10ft.)	dug by	23	= 920
„	(1ft. „ 1ft.)	„	23	$=\frac{920\times(17\times 10)\times 18}{23\times(15\times 12)}$
„	(1ft. „ 1ft.)	„	1	
„	(15ft. „ 12ft.)	„	1	
„	(15ft. „ 12ft.)	„	18	

$$\frac{\overset{20}{\cancel{\overset{40}{\cancel{920}}}}\times 17\times \overset{2}{\cancel{10}}\times \overset{3}{\cancel{18}}}{\cancel{23}\times \underset{5}{\cancel{15}}\times \underset{2}{\cancel{12}}}=680 \text{ feet.}$$

But the 5 men and 3 boys are only at work half the time,

$\therefore\ 680\div 2=340$ ft. *Ans.*

33. Simple Interest.—*Interest is money which is paid by a borrower of money, for the use of the money he borrows.* All problems in Simple Interest are but examples of Rule of Three, either Simple or Compound, under another name. If worked out according to the Unitary Method, children will have no difficulty in understanding what is required, under the various forms in which these questions are presented.

34. After the explanation of the technical terms, *simple interest*, *rate per cent*, *per annum*, *principal*, and *amount*, children will easily see that the two following sums are identically the same, and that the special phraseology alone is the cause of the apparent difference.

(*a*) What must I pay for the rent of 250 acres for 5 years, if I pay £5 for the rent of 100 acres for 1 year?

(*b*) What must I pay for the use of £250 for 5 years, if I pay £5 for the use of £100 for 1 year; or, in other words,

Find the interest on £250 for 5 years at 5 per cent per annum.

The statement, when the interest has to be found, is always—

Interest on £100 for given time = so much.

(Ex. 70.)—Find the simple interest on £85 for a year at 5 per cent.

$$
\begin{array}{lrcl}
 & \text{£} & \text{Year.} & \text{£} \\
\text{Interest on} & 100 \text{ for} & 1 = & 5 \\
\text{,,} & 1 \text{ ,,} & 1 = & \frac{5}{100} \\
\text{,,} & 85 \text{ ,,} & 1 = & \frac{5\times 85}{100}
\end{array}
$$

$$\text{£}\frac{\cancel{5}\times \overset{17}{\cancel{85}}}{\underset{\underset{4}{\cancel{20}}}{\cancel{100}}} = \text{£}\frac{17}{4} = \text{£}4\ 5s. \quad \textit{Ans.}$$

When the Interest for one year only has to be found, the year column may be omitted and the statement be made thus—

$$\text{Interest on } \overset{\text{£}}{100} = \overset{\text{£}}{5}.$$

In these cases the problem is one in Simple Rule of Three. If more than *one* year enters into the demand, the question then becomes an easy one in Compound Rule of Three, the number of years always being the second term in the statement line.

(Ex. 71.)—What is the interest on £267 for 4 years at 5 per cent?

$$\begin{array}{lrlr} & \pounds & \text{Year.} & \pounds \\ \text{Interest on} & 100 & \text{for } 1 & =5 \\ \quad " & 1 & \;"\; 1 & \\ \quad " & 267 & \;"\; 1 & \\ \quad " & 267 & \;"\; 4 & \end{array}\Bigg\} = \frac{5 \times 267 \times 4}{100}$$

$$\pounds\frac{\cancel{5} \times 267 \times \cancel{4}}{\underset{\underset{5}{\cancel{25}}}{\cancel{100}}} = \pounds\frac{267}{5} = \pounds 53\ 8\text{s.} \quad \textit{Ans.}$$

(Ex. 72.)—What is the interest on £964 15s. for 6 years at 4 per cent?

$$\pounds 964\ 15\text{s.} = \pounds 964\tfrac{3}{4}.$$

$$\begin{array}{lrlr} & \pounds & \text{Year.} & \pounds \\ \text{Interest on} & 100 & \text{for } 1 & =4 \\ \quad " & 1 & \;"\; 1 & \\ \quad " & 964\frac{3}{4} & \;"\; 1 & \\ \quad " & 964\frac{3}{4} & \;"\; 6 & \end{array}\Bigg\} = \frac{4 \times 964\frac{3}{4} \times 6}{100}$$

$$\pounds\frac{\frac{4}{1} \times \frac{3859}{4} \times \frac{6}{1}}{\frac{100}{1}} = \frac{\pounds\cancel{4} \times 3859 \times \overset{3}{\cancel{6}}}{\cancel{4} \times \underset{50}{\cancel{100}}} = \frac{\pounds 11577}{50} = \pounds 231\ 10\text{s.}\ 9\tfrac{1}{2}\text{d} + \tfrac{2}{5} \quad \textit{Ans.}$$

(Ex. 73.)—Find the interest on £2368 10s. for $4\frac{1}{2}$ years at $4\frac{1}{2}$ per cent.

$$\pounds 2368\ 10\text{s.} = \pounds 2368\tfrac{1}{2}.$$

$$\begin{array}{lrlr} & \pounds & \text{Year.} & \pounds \\ \text{Interest on} & 100 & \text{for } 1 & =4\frac{1}{2} \\ \quad " & 1 & \;"\; 1 & \\ \quad " & 2368\frac{1}{2} & \;"\; 1 & \\ \quad " & 2368\frac{1}{2} & \;"\; 4\frac{1}{2} & \end{array}\Bigg\} = \frac{4\frac{1}{2} \times 2368\frac{1}{2} \times 4\frac{1}{2}}{100}$$

$$\pounds\frac{\frac{9}{2} \times \frac{4737}{2} \times \frac{9}{2}}{\frac{100}{1}} = \pounds\frac{9 \times 4737 \times 9}{2 \times 2 \times 2 \times 100} = \pounds\frac{42633 \times 9}{8 \times 100} = \pounds\frac{383697}{800}$$

$$= \pounds 479\ 12\text{s.}\ 5\text{d.} + \tfrac{2}{5}\ (\text{or } 5\tfrac{1}{10}\text{d.}). \quad \textit{Ans.}$$

(Ex. 74.)—Required the interest on £768 9s. 6d. for $9\frac{1}{2}$ years at $3\frac{1}{4}$ per cent.

$$\begin{array}{lrrrlll}
 & \pounds & \text{s.} & \text{d.} & & \text{Year.} & \pounds \\
\text{Interest on} & 100 & 0 & 0 & \text{for} & 1 & =3\frac{1}{4} \\
\quad " & 1 & 0 & 0 & " & 1 & \\
\quad " & 768 & 9 & 6 & " & 1 & =\dfrac{3\frac{1}{4}\times\pounds768\text{ 9s. 6d.}\times9\frac{1}{2}}{100} \\
\quad " & 768 & 9 & 6 & " & 9\frac{1}{2} &
\end{array}$$

$$\frac{\pounds3\frac{1}{4}\times\pounds768\text{ 9s. 6d.}\times9\frac{1}{2}}{100}=\frac{\pounds23726\text{ 13s. }3\frac{3}{4}\text{d.}}{100}=\pounds237\text{ 5s. }3\tfrac{3}{4}\text{d.}+\tfrac{99}{100}.\ \textit{Ans.}$$

(Ex. 75.)—Required the simple interest on £460 12s. 6d. for 2 years and 4 months at 5 per cent per annum.

£460 12s. 6d. $=\pounds460\frac{5}{8}$. 2 years 4 months $=2\frac{1}{3}$ years.

$$\begin{array}{lrlll}
 & \pounds & & \text{Year.} & \pounds \\
\text{Interest on} & 100 & \text{for} & 1 & =5 \\
\quad " & 1 & " & 1 & \\
\quad " & 460\frac{5}{8} & " & 1 & =\dfrac{5\times460\frac{5}{8}\times2\frac{1}{3}}{100} \\
\quad " & 460\frac{5}{8} & " & 2\frac{1}{3} &
\end{array}$$

$$\pounds\frac{\frac{5}{1}\times\frac{3685}{8}\times\frac{7}{3}}{\frac{100}{1}}=\pounds\frac{\cancel{5}\times\overset{737}{\cancel{3685}}\times7}{8\times3\times\underset{\underset{4}{\cancel{20}}}{\cancel{100}}}=\pounds\frac{737\times7}{8\times3\times4}=\pounds\frac{5159}{96}$$

$=\pounds53$ 14s. $9\frac{1}{2}$d. *Ans.*

(Ex. 76.)—Find the simple interest upon £41 13s. 4d. for 8 months at $4\frac{1}{2}$ per cent.

£41 13s. 4d. $=\pounds41\frac{2}{3}$

$$\begin{array}{lrlll}
 & \pounds & & \text{Months.} & \pounds \\
\text{Interest on} & 100 & \text{for} & 12 & =4\frac{1}{2} \\
\quad " & 1 & " & 12 & \\
\quad " & 1 & " & 1 & =\dfrac{4\frac{1}{2}\times41\frac{2}{3}\times8}{100\times12} \\
\quad " & 41\frac{2}{3} & " & 1 & \\
\quad " & 41\frac{2}{3} & " & 8 &
\end{array}$$

$$\pounds\frac{\frac{9}{2}\times\frac{125}{3}\times\frac{8}{1}}{\frac{100}{1}\times\frac{12}{1}}=\pounds\frac{\overset{3}{\cancel{9}}\times125\times\overset{2}{\cancel{8}}}{\cancel{2}\times\cancel{3}\times100\times\underset{4}{\cancel{12}}}=\pounds\frac{125}{100}=\pounds1\text{ 5s.}\quad\textit{Ans.}$$

(Ex. 77.)—What is the amount of £968 16s. 7d. for $10\frac{1}{4}$ years at $4\frac{5}{8}$ per cent?

First find the interest; then, interest for the given time plus the original principal is the *amount*.

	£	s.	d.		Year.	£
Interest on	100	0	0	for	1	$= 4\frac{5}{8}$
„	1	0	0	„	1	
„	968	16	7	„	1	$= \frac{4\frac{5}{8} \times £968\ 16s.\ 7d. \times 10\frac{1}{4}}{100}$
„	968	16	7	„	$10\frac{1}{4}$	

$$\frac{£4\frac{5}{8} \times £968\ 16s.\ 7d. \times 10\frac{1}{4}}{100} = \frac{£45928\ 11s.\ 1\frac{7}{8}d.}{100} = £459\ 5s.\ 8\frac{1}{2}d. +$$

$\therefore$ amount $= £968\ 16s.\ 7d. + £459\ 5s.\ 8\frac{1}{2}d. = £1428\ 2s.\ 3\frac{1}{2}d.$ *Ans.*

(Ex. 78.)—What is the amount of £864 for 120 days at $4\frac{1}{2}$ per cent per annum?

	£		Days.	£
Interest on	100	for	365	$= 4\frac{1}{2}$
„	1	„	365	
„	1	„	1	$= \frac{4\frac{1}{2} \times 864 \times 120}{100 \times 365}$
„	864	„	1	
„	864	„	120	

$$£\frac{{}^{*}9 \times \overset{432}{\cancel{864}} \times \overset{\overset{6}{\cancel{24}}}{\cancel{120}}}{{}^{*}\cancel{2} \times \underset{25}{\cancel{100}} \times \underset{73}{\cancel{365}}} = £\frac{432 \times 6 \times 9}{25 \times 73} = £\frac{23328}{1825} = £12\ 15s.\ 7\frac{3}{4}d. + \tfrac{61}{365}.$$

Amount $= £864 + £12\ 15s.\ 7\frac{3}{4}d. + \frac{61}{365} = £876\ 15s.\ 7\frac{3}{4}d. + \frac{61}{365}$. *Ans.*

35. The Three per Cents.—The greater part of Government securities, in which so many people invest their money, bears interest at 3 per cent per annum, paid *half-yearly*. In calculating the interest for half a year, it is generally best to *halve the principal and then calculate as for a year; or, find for a year and then halve the answer.*

(Ex. 79.)—Find the simple interest on £2360, for half a year, at 3 per cent per annum.

---

* The formation of $\frac{9}{2}$ from $4\frac{1}{2}$, and its arrangement in the fraction *at one operation*, ought by this time to be sufficiently familiar to the children to neither occasion difficulty nor be a source of error.

Half principal $= £2360 \div 2 = £1180$.

$$\begin{array}{lrlcc} & £ & \text{Year.} & & £ \\ \text{Interest on} & 100 & \text{for } 1 & = & 3 \\ ,, & 1 & ,,\ 1 & = & \frac{3}{100} \\ ,, & 1180 & ,,\ 1 & = & \frac{3 \times 1180}{100} \end{array}$$

$$£\frac{3 \times 1180}{100} = £\frac{3540}{100} = £35\ 8s. \quad \textit{Ans.}$$

36. Calculation of Rate, Time, and Principal.—In addition to the simple case of calculation of Interest at a certain rate and for a given time, there are *three* other cases which require attention.

(1) Calculation of *Rate* per cent; Principal, Amount, (or total Interest) and Time being given.

(2) Calculation of *Time*; Principal, Rate and Amount (or total Interest) being given.

(3) Calculation of *Principal*; Amount (or total Interest) Rate and Time being given.

(Ex. 80.)—In two years £250 put out at simple interest amounts to £302 10s. What is the rate per cent per annum?

Here *the demand* is the rate of interest on £100 for 1 year.

£302 10s. − £250 = £52 10s. = interest for 2 years.

$$\begin{array}{lrlcl} & £ & \text{Year.} & & £ \\ \text{Interest on} & 250 & \text{for } 2 & = & 52\tfrac{1}{2} \\ ,, & 1 & ,,\ 2 & \multirow{3}{*}{$\Bigg\}=$} & \multirow{3}{*}{$\frac{52\frac{1}{2} \times 100}{250 \times 2}$} \\ ,, & 1 & ,,\ 1 & & \\ ,, & 100 & ,,\ 1 & & \end{array}$$

$$£\frac{{}^*105 \times \overset{\overset{2}{\cancel{40}}}{\cancel{100}}}{{}^*\cancel{2} \times \underset{10}{\cancel{250}} \times \cancel{2}} = £\frac{105}{10} = £10\tfrac{1}{2} \text{ per cent.} \quad \textit{Ans.}$$

* See Note to Ex. 78.

(Ex. 81.)—The interest on £475 for a year amounted to £17 2s., what was the rate per cent.?

$$£17\ 2s. = £17\tfrac{1}{10}.$$

$$\begin{array}{lrl} & \overset{£}{} & \\ \text{Interest on} & \overset{£}{475} = & \overset{£}{17\tfrac{1}{10}} \\ \quad " & 1 = & \dfrac{17\frac{1}{10}}{475} \\ \quad " & 100 = & \dfrac{17\frac{1}{10} \times 100}{475} \end{array}$$

$$£\frac{17\frac{1}{10} \times 100}{475} = £\frac{\overset{9}{\cancel{171}} \times \overset{\overset{2}{\cancel{4}}}{\cancel{100}}}{\underset{5}{\cancel{10}} \times \underset{\cancel{19}}{\cancel{475}}} = £\frac{18}{5} = £3\tfrac{3}{5} \text{ per cent.} \quad \textit{Ans.}$$

(Ex. 82.)—If £880 amounts to £899 5s. at simple interest for $\frac{7}{12}$ of a year, find the rate per cent. per annum.

Here we may first find how much interest £880 gains in a year, and then the rate by another operation.

£899 5s. − £880 = £19 5s. = £19¼ = interest on £880 for $\frac{7}{12}$ of a year

$$\begin{array}{lrl} \text{Interest for} & \overset{\text{Year.}}{\tfrac{7}{12}} = & \overset{£}{19\tfrac{1}{4}} \\ \quad " & 1 = & \dfrac{19\frac{1}{4}}{\frac{7}{12}} \end{array}$$

$$\frac{£19\frac{1}{4}}{\frac{7}{12}} = £\frac{\frac{77}{4}}{\frac{7}{12}} = £\frac{\overset{11}{\cancel{77}} \times \overset{3}{\cancel{12}}}{\cancel{4} \times \cancel{7}} = £33 \text{ interest on £880 for 1 year.}$$

Then—

$$\begin{array}{lrl} \text{Interest on} & \overset{£}{880} = & \overset{£}{33} \\ \quad " & 1 = & \dfrac{33}{880} \\ \quad " & 100 = & \dfrac{33 \times 100}{880} \end{array}$$

$$£\frac{\overset{3}{\cancel{33}} \times \overset{\overset{5}{\cancel{10}}}{\cancel{100}}}{\underset{\underset{\underset{4}{\cancel{44}}}{\cancel{88}}}{\cancel{880}}} = £\frac{3 \times 5}{4} = £\frac{15}{4} = £3\tfrac{3}{4} \text{ per cent.} \quad \textit{Ans.}$$

Or, by a neater method at one operation—

$$\begin{array}{llll} & \pounds & \text{Year.} & \pounds \\ \text{Interest on} & 880 & \text{for } \frac{7}{12} & =19\frac{1}{4} \\ \quad ,, & 1 & ,,\ \frac{7}{12} & \multirow{3}{*}{$\left.\right\}\dfrac{19\frac{1}{4}\times 100}{880\times\frac{7}{12}}$} \\ \quad ,, & 1 & ,,\ 1 & \\ \quad ,, & 100 & ,,\ 1 & \end{array}$$

$$\pounds\frac{\frac{77}{4}\times\frac{100}{1}}{\frac{880}{1}\times\frac{7}{12}}=\pounds\frac{\overset{11}{\cancel{77}}\times\overset{5}{\cancel{100}}\times\overset{3}{\cancel{12}}}{\cancel{4}\times\underset{\underset{4}{\cancel{40}}}{\cancel{880}}\times\cancel{7}}=\pounds\frac{15}{4}=\pounds3\tfrac{3}{4}\text{ per cent.}\quad \textit{Ans.}$$

(Ex. 83.)—In what time will the interest on £360 amount to £144 at 5 per cent per annum?

Here we may first find how much interest £360 gains in a year, and then, by a second step, find the time.

$$\begin{array}{lrl} & \pounds & \pounds \\ \text{Interest on} & 100 & =5 \\ \quad ,, & 1 & =\frac{5}{100} \\ \quad ,, & 360 & =\frac{5\times 360}{100} \end{array}$$

$$\pounds\frac{\cancel{5}\times\overset{18}{\cancel{360}}}{\underset{\cancel{20}}{\cancel{100}}}=\pounds18$$

Then—

$$\begin{array}{lrll} & \pounds & & \text{Year.} \\ \text{Time to gain} & 18 & \text{interest} & =1 \\ \quad ,, & 1 & \quad ,, & =\frac{1}{18} \\ \quad ,, & 144 & \quad ,, & =\frac{1\times 144}{18} \end{array}$$

$$\frac{1\times 144}{18}=8\text{ years.}\quad \textit{Ans.}$$

Or, briefer—

Since £18 is gained in 1 year, £144 is gained in £144 ÷ £18 = 8 years. *Ans.*

Or, by a neater method at one operation—

$$\begin{array}{lrlrl} & \pounds & & \pounds & \text{Year.} \\ \text{Time for} & 100 & \text{to gain} & 5 & =1 \\ \text{,,} & 1 & \text{,,} & 5 & \\ \text{,,} & 1 & \text{,,} & 1 & \\ \text{,,} & 360 & \text{,,} & 1 & \\ \text{,,} & 360 & \text{,,} & 144 & \end{array} \left.\vphantom{\begin{array}{c}1\\1\\1\\1\end{array}}\right\} = \frac{1 \times 100 \times 144}{5 \times 360}$$

$$\frac{1 \times \overset{20}{\cancel{100}} \times 144}{\cancel{5} \times \underset{18}{\cancel{360}}} = \frac{144}{18} = 8 \text{ years.} \quad \textit{Ans.}$$

(Ex. 84.)—In what time will £225 amount to £256 10s at $3\frac{1}{2}$ per cent per annum, simple interest?

£256 10s. − £225 = £31 10s. = £$31\frac{1}{2}$ the amount of interest to be obtained at $3\frac{1}{2}$ per cent per annum; then by Compound Proportion—

$$\begin{array}{lrlrl} & \pounds & & \pounds & \text{Year.} \\ \text{Time for} & 100 & \text{to gain} & 3\frac{1}{2} & =1 \\ \text{,,} & 1 & \text{,,} & 3\frac{1}{2} & \\ \text{,,} & 1 & \text{,,} & 1 & \\ \text{,,} & 225 & \text{,,} & 1 & \\ \text{,,} & 225 & \text{,,} & 31\frac{1}{2} & \end{array} \left.\vphantom{\begin{array}{c}1\\1\\1\\1\end{array}}\right\} = \frac{1 \times 100 \times 31\frac{1}{2}}{3\frac{1}{2} \times 225}$$

$$\frac{1 \times 100 \times 31\frac{1}{2}}{3\frac{1}{2} \times 225} = \frac{\overset{4}{\cancel{100}} \times \overset{9}{\cancel{63}} \times \cancel{2}}{\cancel{7} \times \underset{9}{\cancel{225}} \times \cancel{2}} = 4 \text{ years.} \quad \textit{Ans.}$$

(Ex. 85.)—In what time will £250 double itself at $2\frac{1}{2}$ per cent per annum simple interest?

To double itself £250 must gain £250 as interest; therefore, by Compound Proportion—

$$\begin{array}{lrlrl} & \pounds & & \pounds & \text{Year.} \\ \text{Time for} & 100 & \text{to gain} & 2\frac{1}{2} & =1 \\ \text{,,} & 1 & \text{,,} & 2\frac{1}{2} & \\ \text{,,} & 1 & \text{,,} & 1 & \\ \text{,,} & 250 & \text{,,} & 1 & \\ \text{,,} & 250 & \text{,,} & 250 & \end{array} \left.\vphantom{\begin{array}{c}1\\1\\1\\1\end{array}}\right\} = \frac{1 \times 100 \times 250}{2\frac{1}{2} \times 250}$$

$$\frac{1 \times 100 \times \cancel{250}}{2\frac{1}{2} \times \cancel{250}} = \frac{100}{2\frac{1}{2}} = 40 \text{ years.} \quad \textit{Ans.}$$

(Ex. 86.)—What principal will amount to £1000 in 5 years at 5 per cent per annum?

By one step we must find the amount of £100 for the given time at the given rate, and then, knowing the principal from which this amount arises, we can find the principal demanded by the question.

Interest on 100 for 5 years @ 5% = £5 × 5 = £25.

∴ Amount of " " " = £100 + £25 = £125.

$$\begin{array}{lll} \text{Principal from which } \overset{\pounds}{125} \text{ amounts} & = & \overset{\pounds}{100} \\ \quad " \quad " \quad 1 \quad " & = & \dfrac{100}{125} \\ \quad " \quad " \quad 1000 \quad " & = & \dfrac{100 \times 1000}{125} \end{array}$$

$$\pounds\frac{100 \times \overset{8}{\cancel{1000}}}{\cancel{125}} = \pounds 100 \times 8 = \pounds 800. \quad \textit{Ans.}$$

(Ex. 87.)—What sum will amount to £580 in 4 years at 5 per cent per annum?

Interest on £100 for 4 years @ 5% = £5 × 4 = £20.

∴ Amount of " " " = £100 + £20 = £120.

$$\begin{array}{lll} \text{Principal from which } \overset{\pounds}{120} \text{ amounts} & = & \overset{\pounds}{100} \\ \quad " \quad " \quad 1 \quad " & = & \dfrac{100}{120} \\ \quad " \quad " \quad 580 \quad " & = & \dfrac{100 \times 580}{120} \end{array}$$

$$\pounds\frac{\overset{5}{\cancel{100}} \times \overset{290}{\cancel{580}}}{\underset{\underset{3}{\cancel{6}}}{\cancel{120}}} = \pounds\frac{290 \times 5}{3} = \pounds\frac{1450}{3} = \pounds 483\ 6s.\ 8d. \quad \textit{Ans.}$$

37. Compound Interest.—Compound interest is money which is paid, not only for the use of the sum lent, but also for the use of the interest on that sum as it becomes due.

38. All compound interest problems resolve themselves into finding the interest for *each separate year* during a certain number of years, the principal, on which this interest is calculated, being increased, each year, by the amount of interest gained in the previous year. The "Method of Unity" can therefore be easily applied in every case.

39. When the calculation is for a considerable number of years, the working becomes somewhat cumbrous, and though it may be greatly shortened by the use of decimals, as shown in Example 88*b* (p. 65), the method exemplified in Example 89 (p. 66) is much to be preferred in ordinary working, when it can be applied.

(Ex. 88.)—Find the compound interest and amount of £1650 for 3 years at 5 per cent per annum.

(a) Here the interest for each of the three years must be found *separately*, the total amount of interest being the sum of the three results.

$$\text{Interest on } \overset{£}{100} = \overset{£}{5}$$

$$\text{,,} \quad 1 = \frac{5}{100}$$

$$\text{,,} \quad 1650 = \frac{5 \times 1650}{100}$$

$$£\frac{\cancel{5} \times \overset{165}{\cancel{1650}}}{\underset{\underset{2}{\cancel{20}}}{\cancel{100}}} = £\frac{165}{2} = £82\ 10s. \quad \text{Interest for 1st year.}$$

£	s.	
1650	0	Original principal.
82	10	1st year's interest.
1732	10	2nd year's principal.

	£	s.		£
Interest on	100	0	=	5
"	1	0	=	$\frac{5}{100}$
"	1732	10	=	$\frac{5 \times £1732\ 10s.}{100}$

$$\frac{£\cancel{5} \times £1732\ 10s.}{\underset{20}{\cancel{100}}} = \frac{£1732\ 10s.}{20} = £86\ 12s.\ 6d. \text{ Interest for 2nd year.}$$

£	s.	d.	
1732	10	0	2nd year's principal.
86	12	6	2nd year's interest.
£1819	2	6	3rd year's principal.

	£	s.	d.		£
Interest on	100	0	0	=	5
"	1	0	0	=	$\frac{5}{100}$
"	1819	2	6	=	$\frac{5 \times £1819\ 2s.\ 6d.}{100}$

$$\frac{£\cancel{5} \times £1819\ 2s.\ 6d.}{\underset{20}{\cancel{100}}} = \frac{£1819\ 2s.\ 6d.}{20} = £90\ 19s.\ 1\tfrac{1}{2}d. \text{ Interest for 3rd yr.}$$

Then —

£	s.	d.	
82	10	0	1st year's interest.
86	12	6	2nd "
90	19	1½	3rd "
260	1	7½	Total interest.
1650	0	0	Original principal.
£1910	1	7½	Amount. *Ans.*

(*b*) **Using Decimals** in the same example and noting, from the formation of the fractions, that, to find the interest of a certain sum at a given rate per cent, we multiply by the rate and divide by 100, the work is much simplified. To divide by 100, move the decimal point *two places to the left.*

$$\text{Interest on } \overset{\pounds}{100} = \overset{\pounds}{5}$$

$$\text{,,} \qquad 1 = \frac{5}{100}$$

$$\text{,,} \qquad 1650 = \frac{5 \times 1650}{100}$$

$\frac{\pounds 5 \times 1650}{100} = \pounds\frac{8250}{100}$	$= \pounds 82{\cdot}5$	Interest for 1st year.
	1650	Original principal.
	1732·5	Principal for 2nd year.
	5	Multiply by rate %.
Dividing by 100	8662·5	
We get	86·625	Interest for 2nd year.
	1732·5	Principal for 2nd year.
	1819·125	Principal for 3rd year
	5	Multiply by rate %.
Dividing by 100	9095·625	
We get	90·95625	Interest for 3rd year.
	1819·125	Principal for 3rd year.
	£1910·08125	Amount, which when the
	20	decimal is reduced, =
		£1910 1s. 7½d. *Ans.*
	1·62500	
	12	
	7·500	
	4	
	2·0	

£	
82·5	
86·625	
90·95625	
£260·08125	Total interest, which when the
20	decimal is reduced, =
1·62500	£260 1s. 7½d. *Ans.*
12	
7·500	
4	
2·0	

(Ex. 89.) **Aliquot Parts.**—Still using the same example, it will be seen that at 5 per cent. the rate is $\frac{1}{20}$ of £100; for, multiplying the principal by 5 and dividing by $100 = \frac{5}{100}$, or $\frac{1}{20}$ of the principal. The following easy method is the one usually employed in all cases where the rate per cent. is a simple aliquot part of £100, as 10 per cent. $= \frac{1}{10}$; $8\frac{1}{3}$ per cent. $= \frac{1}{12}$; 5 per cent. $= \frac{1}{20}$; 4 per cent. $= \frac{1}{25}$; $2\frac{1}{2}$ per cent. $= \frac{1}{40}$; 2 per cent. $= \frac{1}{50}$.

	£	
£5 = $\frac{1}{20}$	1650	
	82·5	Interest for 1st year.
£5 = $\frac{1}{20}$	1732·5	Principal for 2nd year.
	86·625	Interest for 2nd year.
£5 = $\frac{1}{20}$	1819·125	Principal for 3rd year.
	90·95625	Interest for 3rd year.
	£1910·08125	Amount, which as before = £1910 1s. 7½d.

40. Calculation for Parts of a Year.—When the Compound Interest for any number of entire years *and a part of a year* is required, it is usual to calculate the

interest for the whole year (of which the part only is wanted), and then take the required part of that year's interest. Thus in Exercise 90 the interest is calculated for 3 years, and $\frac{2}{3}$ of the third year's interest is taken for the 8 months.

(Ex. 90.)—What is the compound interest on £75 10s. for 2 years and 8 months at 5 per cent per annum?

£75 10s. = £75·5

	£	
$5\% = \frac{1}{20}$	75·5	
	3·775	Interest for 1st year.
$5\% = \frac{1}{20}$	79·275	Principal for 2nd year.
	3·96375	Interest for 2nd year.
$5\% = \frac{1}{20}$	83·23875	Principal for 3rd year.
	4·1619375	Interest for 3rd year.
	2	
3	8·3238750	
	2·774625	Interest for 8 months ($\frac{2}{3}$ of 3rd year.)

	£	
Then	3·775	
	3·96375	
	2·774625	
	£10·513375	Total interest, which when
	20	the decimal is reduced =
		£10 10s. 3·21d. *Ans.*
	10·267500	
	12	
	3·2100	

41. Amount at Compound Interest.—In cases where the *amount* of a sum of money, put out at Compound Interest, is required for a number of entire years, the brief and neat method now to be explained should be employed.*

* See "A Treatise on Arithmetic," by Hamblin Smith (Rivingtons), page 191. The author has obtained many valuable hints from this excellent book.

Suppose the rate per cent to be £5 per annum, then—

Amount of £100 at end of 1st year = £105.

And amount of £1 at end of 1st year $= \frac{105}{100}$ of £1.

And amount of any sum at end of 1st year $= \frac{105}{100}$ of that sum.

Again—

Amount of £100 at end of 2nd year $= \frac{105}{100}$ of 1st year's amount.

$\therefore$ Amount of any sum at end of 2nd year $= \frac{105}{100}$ of $\frac{105}{100}$ of original principal.

Again—

Amount of any sum at end of 3rd year $= \frac{105}{100}$ of second year's amount.

$\therefore$ Amount of any sum at end of 3rd year $= \frac{105}{100}$ of $\frac{105}{100}$ of $\frac{105}{100}$ of original principal,

and so on for any number of years. Hence, to find the amount at Compound Interest, Multiply the principal by the fraction, which represents the amount of £1 at the given rate, as many times as there are years to be calculated, and divide as many times by 100.

(Ex. 91.) Find the amount of £3745 for 3 years at 5 per cent per annum, Compound Interest.

Here the amount will be—

$$\frac{105}{100} \text{ of } \frac{105}{100} \text{ of } \frac{105}{100} \text{ of £3745, for—}$$

	£		Year.		£	£
Amount of	100	for	1	at	5%	$= 105$
„	1	„	1	„		$= \frac{105}{100}$
„	1	„	2	„		$= \frac{105}{100} \times \frac{105}{100}$
„	1	„	3	„		$= \frac{105}{100} \times \frac{105}{100} \times \frac{105}{100}$
„	3745	„	3	„		$= \frac{105}{100} \times \frac{105}{100} \times \frac{105}{100} \times 3745$

∴ Multiply £3745 three times by 105, and divide the result three times by 100. Dividing three times by 100 is simply marking off six decimal places in the final product, for $100 \times 100 \times 100 = 1000000$.

```
                        £
                      3745
                       105*
                    ------
                     18725
                    37450
                    ------
                    393225
                       105
                    ------
                   1966125
                  3932250
                  --------
                  41288625
                       105
                  --------
                 206443125
                412886250
Dividing by 100 } ----------
3 times we get  } £4335·305625 amount required, which, when the
                          20      decimal is reduced, =
                  ---------
                  ·6112500              £4335 0s. 7·335d.  Ans.
                        12
                  -------
                  7·33500
```

* For a neat method (useful for Pupil-Teachers' work) of multiplying by such numbers as 102, 103, 104, 105, &c., see Hamblin Smith's Arithmetic, p. 192.

(Ex. 92.)—Find the amount of £475 15s. for two years at 3 per cent per annum, Compound Interest.

Here the amount will be—

$$\frac{103}{100} \text{ of } \frac{103}{100} \text{ of } £475{\cdot}75\text{, for—}$$

	£		Year.		£	£
Amount of	100	for	1	at	3%	$=103$
„	1	„	1	„		$=\frac{103}{100}$
„	1	„	2	„		$=\frac{103}{100}\times\frac{103}{100}$
„	475·75	„	2	„		$=\frac{103}{100}\times\frac{103}{100}\times 475{\cdot}75$

$$£\frac{103}{100}\times\frac{103}{100}\times 475{\cdot}75=£\frac{504723175}{10000}=£504{\cdot}723175$$

$$=£504\ 14\text{s.}\ 5{\cdot}562\text{d.}\quad \textit{Ans.}$$

(Ex. 93.)—In three years a sum of money amounts, with Compound Interest at 5 per cent, to £926 2s. What will it amount to in 5 years?

At 5 per cent interest $=\frac{5}{100}$ of principal.

Interest for 4th year $=£926\frac{1}{10}\times\frac{5}{100}$

$$=\frac{£9261}{\cancel{10}_{\,2}}\times\frac{\cancel{5}}{100}=\frac{£9261}{200}=£46\frac{61}{200}$$

Principal for 5th year $=£926\frac{1}{10}+£46\frac{61}{200}=£972\frac{81}{200}$.

Interest „ $=£972\frac{81}{200}\times\frac{5}{100}$

$$=\frac{£194481}{\cancel{200}_{\,40}}\times\frac{\cancel{5}}{100}=\frac{£194481}{4000}=£48\frac{2481}{4000}$$

Amount $=£972\frac{81}{200}+£48\frac{2481}{4000}=£1021\frac{101}{4000}=£1021\ 0\text{s.}\ 6\text{d.}+\frac{3}{50}$.
Ans.

(Ex. 94.)—My income is derived from the proceeds of £4,550 at a certain rate per cent, and £5,420 at one per cent more than the former. If my whole income is £453, find the rate of interest.

Total amount invested $= £4550 + £5420 = £9970$.

One per cent interest on $£5420 = £54\frac{1}{5}$.

Income from £9970 at the lower rate $= £453 - £54\frac{1}{5} = £398\frac{4}{5}$.

$$\text{Income from } \overset{£}{9970} = \overset{£}{398\tfrac{4}{5}}$$

$$\text{,,} \qquad 1 = \frac{398\frac{4}{5}}{9970}$$

$$\therefore \qquad \text{,,} \qquad 100 = \frac{398\frac{4}{5} \times 100}{9970}$$

$$£\frac{398\frac{4}{5} \times 100}{9970} = £\frac{1994 \times 10\cancel{0}}{997\cancel{0} \times 5} = £\frac{19940}{4985} = £4\%$$

Hence the lower rate is 4% and the higher one 5%. *Ans.*

42. Percentages.*—As a convenient standard of comparison, the number 100 is used in many transactions of every-day life. The hundred referred to may be either £100, 100s., 100d., 100 persons, 100oz., 100lbs., &c., and the number representing the percentage is so many units of this hundred. It is usual to consider the 100 as an *abstract* number; thus, talking of money, we generally say five per cent, and not five pounds per cent, though there is no reason why a concrete number should not be spoken of, and, in fact, it often makes a problem plainer to consider the percentage as concrete rather than abstract.

43. All problems included under the general terms of *Percentages* and *Profit and Loss* reduce themselves to simple cases of Rule of Three, and common-sense reasoning will enable every kind of problem to be easily solved. In cases where the gain or loss per cent. has to be calculated on *business transactions,* care must be taken to observe whether the percentage has to be calculated on the *selling* price or on the *cost* price of the goods. That this is important will be seen from the following examples :—

* Latin *per centum*, abbreviated to per cent. = so much *for a hundred.*

(Ex. 95.)—A tradesman marks various goods so as to gain (*a*) 25 per cent. ; (*b*) 20 per cent. ; (*c*) 10 per cent. ; and (*d*) 5 per cent. ; what fractional part in each case is profit ?

Here the first question to be asked is, "Does the tradesman calculate his gain on the cost price or on the selling price ?"

(*a*) Gain on cost price of $100 = \frac{100}{25} = 4$. Therefore $\frac{1}{4}$ of *cost* is profit.

Gain on selling price of $100 = 100$ cost $+ 25$ gain $= \frac{125}{25} = 5$. Therefore $\frac{1}{5}$ of *selling* price is profit.

(*b*) Gain on cost price of $100 = \frac{100}{20} = 5$. Therefore $\frac{1}{5}$ of *cost* is profit.

Gain on selling price of $100 = 100$ cost $+ 20$ gain $= \frac{120}{20} = 6$. Therefore $\frac{1}{6}$ of *selling* price is profi:

(*c*) Gain on cost price of $100 = \frac{100}{10} = 10$. Therefore $\frac{1}{10}$ of *cost* is profit.

Gain on selling price of $100 = 100$ cost $+ 10$ gain $= \frac{110}{10} = 11$ Therefore $\frac{1}{11}$ of *selling* price is profit.

(*d*) Gain on cost price of $100 = \frac{100}{5} = 20$. Therefore $\frac{1}{20}$ of *cost* is profit.

Gain on selling price of $100 = 100$ cost $+ 5$ gain $= \frac{105}{5} = 21$ Therefore $\frac{1}{21}$ of *selling* price is profit.

44. Percentage must always be calculated on the *cost price*, unless there is something in the question which indicates that the selling price should be taken.

(Ex. 96.)—Cloth is bought at 12s. a yard, and sold at 16s. a yard. Find the gain per cent.

Gain on cost price of one yard = 16s. − 12s. = 4s.

Then—

$$\begin{array}{lrl} & \text{Shil.} & \text{Shil.} \\ \text{Gain on} & 12 = & 4 \\ \quad ,, & 1 = & \frac{4}{12} \\ \quad ,, & 100 = & \frac{4 \times 100}{12} \end{array}$$

$$\frac{\cancel{4} \times 100}{\underset{3}{\cancel{12}}} = \frac{100}{3} = 33\tfrac{1}{3}\%. \quad \textit{Ans.}$$

This answer means that on the cost price of 100d., $33\frac{1}{3}$ pence is gained, *i.e.*, the cloth would be sold for $133\frac{1}{3}$d. If cost be 100s., then gain is $33\frac{1}{3}$s., if £100 the gain is £$33\frac{1}{3}$.

(Ex. 97.)—If tea be sold at 3s. 4d. per lb., which cost 2s. 9d. per lb.; what is the gain per cent?

Gain per lb. on cost price = 3s. 4d. − 2s. 9d. = 7d. 2s. 9d. = 33d.

$$\begin{array}{lrl} & \text{Pence.} & \text{Pence.} \\ \text{Gain on} & 33 = & 7 \\ \quad ,, & 1 = & \frac{7}{33} \\ \quad ,, & 100 = & \frac{7 \times 100}{33} \end{array}$$

$$\frac{7 \times 100}{33} = \frac{700}{33} = 21\tfrac{7}{33} \text{ per cent.} \quad \textit{Ans.}$$

(Ex. 98.)—A man sells a horse for £28, thereby gaining 8 per cent; what did it cost him?

Since the £28 includes cost price + gain, the cost price will be less than £28. Now, £100 sold to gain 8 per cent = £108; ∴ if cost price of £108 = £100, what will it be for £28?

$$\text{Cost price of } \overset{\pounds}{108} = \overset{\pounds}{100}$$

$$\text{"} \quad 1 = \frac{100}{108}$$

$$\text{"} \quad 28 = \frac{100 \times 28}{108}$$

$$\frac{\pounds 100 \times \overset{7}{\cancel{28}}}{\underset{27}{\cancel{108}}} = \pounds\frac{700}{27} = \pounds 25\ 18s.\ 6\tfrac{2}{3}d. \quad \textit{Ans.}$$

(Ex. 99.)—A man sells a horse, which cost him £28, at a loss of seven per cent. What did he get for it?

Here what cost £100 would be sold for £100 − £7 = £93, ∴ what would £28 be sold for?

$$\text{Selling price of } \overset{\pounds}{100} = \overset{\pounds}{93}.$$

$$\text{"} \quad 1 = \frac{93}{100}$$

$$\text{"} \quad 28 = \frac{93 \times 28}{100}$$

$$\frac{\pounds 93 \times \overset{7}{\cancel{28}}}{\underset{25}{\cancel{100}}} = \frac{\pounds 651}{25} = \pounds 26\ 0s.\ 9\tfrac{3}{5}d. \quad \textit{Ans.}$$

(Ex. 100.)—A grocer uses instead of a 1lb. weight one that only weighs 15·75oz. What does he gain per cent by his dishonesty?

On 1lb., weighed by the light weight, gain = 16oz. − 15·75oz. = ·25oz.

$$\text{Gain on } \overset{oz.}{15{\cdot}75} = \overset{oz.}{{\cdot}25}$$

$$\text{"} \quad 1 = \frac{{\cdot}25}{15{\cdot}75}$$

$$\text{"} \quad 100 = \frac{{\cdot}25 \times 100}{15{\cdot}75}$$

$$\frac{{\cdot}25 \times 100}{15{\cdot}75} = \frac{25}{15{\cdot}75} = \frac{2500}{1575} = 1\tfrac{37}{63} \text{ per cent.} \quad \textit{Ans}$$

(Ex. 101.)—If cheese, sold at $8\frac{1}{2}$d. per lb., gives a profit of seven per cent, what was the cost price?

Here selling price of what cost 100d. = 107d.:
$\therefore$ cost price of 107d. = 100d.

$$\begin{aligned} \text{Cost price of } \overset{\text{d.}}{107} &= \overset{\text{d.}}{100} \\ \text{,,} \qquad 1 &= \frac{100}{107} \\ \text{,,} \qquad 8\tfrac{1}{2} &= \frac{100 \times 8\frac{1}{2}}{107} \end{aligned}$$

$$\frac{100 \times 8\frac{1}{2}}{107} = \frac{\overset{50}{\cancel{100}} \times 17}{107 \times \cancel{2}} = \frac{850}{107} = 7\tfrac{101}{107}\text{d. or } 7\tfrac{3}{4}\text{d.} + \tfrac{83}{107}. \quad \textit{Ans.}$$

(Ex. 102.) A man bought a book for 7s. 6d., and sold it at a profit of 15 per cent. How much did he get for it?

$$\begin{aligned} \text{Selling price of } \overset{\text{Shill.}}{100} &= \overset{\text{Shill.}}{115} \\ \text{,,} \qquad 1 &= \frac{115}{100} \\ \text{,, } 7/6 \text{ (or } 7\tfrac{1}{2}\text{s.)} &= \frac{115 \times 7\frac{1}{2}}{100} \end{aligned}$$

$$\frac{115 \times 7\frac{1}{2}}{100} = \frac{\overset{23}{\cancel{115}} \times \overset{3}{\cancel{15}}}{\underset{\underset{4}{\cancel{20}}}{\cancel{100}} \times 2} = \frac{69}{8} = 8/7\tfrac{1}{2}. \quad \textit{Ans.}$$

(Ex. 103.) By selling nuts at 5d. per lb. a loss of 20 per cent. results; at what price must they be sold so as to gain 20 per cent?

Selling price of 100d. = 100d. − 20d. = 80d.

$$\begin{aligned} \text{Cost price of nuts sold for } \overset{\text{d.}}{80} &= \overset{\text{d.}}{100} \\ \text{,,} \qquad \text{,,} \qquad 1 &= \frac{100}{80} \\ \text{,,} \qquad \text{,,} \qquad 5 &= \frac{100 \times 5}{80} \end{aligned}$$

$$\frac{\overset{5}{\cancel{100}} \times 5}{\underset{4}{\cancel{80}}} = \frac{25}{4} = 6\tfrac{1}{4}\text{d. per lb. cost.}$$

Cost price per lb. being $6\frac{1}{4}$d., the selling price, to gain 20 per cent., may be found.

$$\text{Selling price of } \overset{\text{d.}}{100} = \overset{\text{d.}}{120}$$

$$\text{,,} \qquad 1 = \frac{120}{100}$$

$$\text{,,} \qquad 6\tfrac{1}{4} = \frac{120 \times 6\frac{1}{4}}{100}$$

$$\frac{120 \times 6\frac{1}{4}}{100} = \frac{\overset{30}{\cancel{120}} \times \cancel{25}}{\underset{4}{\cancel{100}} \times \cancel{4}} = \frac{30}{4} = 7\tfrac{1}{2}\text{d. per lb.} \quad \textit{Ans.}$$

But this calculation can be made at one operation.

Selling price in first case $= 100\text{d.} - 20\text{d.} = 80\text{d.}$

,, second ,, $= 100\text{d.} + 20\text{d.} = 120\text{d.}$

$\therefore$ if selling price in first case be 5d., what must it be in second case.

Selling price at 80 per cent. of cost $= 5$d.

$$\text{,,} \quad 1 \quad \text{,,} \quad = \frac{5}{80}$$

$$\text{,,} \quad 120 \quad \text{,,} \quad = \frac{5 \times 120}{80}$$

$$\frac{5 \times \overset{3}{\cancel{120}}}{\underset{2}{\cancel{80}}} = \frac{15}{2}\text{d.} = 7\tfrac{1}{2}\text{d.} \quad \textit{Ans.}$$

(Ex. 104.)—By selling tea at 5s. 4d. per lb., a grocer clears one-eighth of his outlay. He then raises the price to 6s. 2d. What is his gain per cent on his outlay now.

$\frac{1}{8}$ % gain $= \frac{100}{8} = 12\frac{1}{2}\%$, $\therefore$ in first case, what cost 100d. sells for $112\frac{1}{2}$d.

5s. 4d. $= 64$d. 6s. 2d. $= 74$d.

$$\text{Selling price of 100d. worth at } \overset{\text{d.}}{64} \text{ per lb.} = \overset{\text{Pence.}}{112\tfrac{1}{2}}$$

$$\text{,,} \qquad \text{,,} \qquad 1 \quad \text{,,} \quad = \frac{112\frac{1}{2}}{64}$$

$$\text{,,} \qquad \text{,,} \qquad 74 \quad \text{,,} \quad = \frac{112\frac{1}{2} \times 74}{64}$$

$$\frac{112\frac{1}{2}\times 74}{64}=\frac{225\times \overset{37}{\cancel{74}}}{\cancel{2}\times 64}=\frac{8325}{64}=130\tfrac{5}{64}$$

∴ $130\frac{5}{64}$ selling price − 100 cost price = $30\frac{5}{64}$ % gain on outlay.

Or, by another method.

Since gain is $\frac{1}{8}$ of outlay, 64d. = $\frac{8}{8}+\frac{1}{8}=\frac{9}{8}$ outlay.

$$\therefore \text{ outlay}=\frac{64}{\frac{9}{8}}\text{d.}=\frac{64\times 8}{9}=56\tfrac{8}{9}\text{d.}$$

Then gain on one lb. in first case = 64d. − $56\frac{8}{9}=7\frac{1}{9}$d.

,, second case = 74d. − $56\frac{8}{9}=17\frac{1}{9}$d.

	d.		Per cent.
Gain of	$7\frac{1}{9}$	per lb.	$=12\frac{1}{2}$
,,	1	,,	$=\dfrac{12\frac{1}{2}}{7\frac{1}{9}}$
,,	$17\frac{1}{9}$	,,	$=\dfrac{12\frac{1}{2}\times 17\frac{1}{9}}{7\frac{1}{9}}$

$$\frac{12\frac{1}{2}\times 17\frac{1}{9}}{7\frac{1}{9}}=\frac{25\times \overset{77}{\cancel{154}}\times \cancel{9}}{\cancel{2}\times \cancel{9}\times 64}=\frac{1925}{64}=30\tfrac{5}{64}\%\text{ gain.}\quad \textit{Ans.}$$

(Ex. 105.)—A man buys goods and sells them at such a price that he receives, for $\frac{4}{7}$ of this price, sufficient to pay for the goods. Find his gain per cent?

If goods are sold for 1s., then $\frac{4}{7}$s. will be cost price.

$$\therefore 1\text{s.}-\tfrac{4}{7}\text{s.}=\frac{7-4}{7}=\tfrac{3}{7}\text{s. gain}$$

That is, gain on $\frac{4}{7}$s. = $\frac{3}{7}$s.; or, multiply by 7^2 to get rid of fractions.

Gain on 28s. = 21s., ∴ what will it be on 100s.?

	Shill.	Shill.
Gain on	28	= 21
,,	1	$=\dfrac{21}{28}$
,,	100	$=\dfrac{21\times 100}{28}$

$$\frac{\overset{3}{\cancel{21}}\times 100}{\underset{4}{\cancel{28}}}=\frac{300}{4}=75\text{ \% gain on outlay.}\quad \textit{Ans.}$$

(Ex. 106.)—If 1 ton 3 cwt. cost £23, what must be the retail price per cwt. so as to gain 5 per cent?

Here, what costs £100 must be retailed for £105 $\therefore$ what must £23 be sold for? But 1 ton 3 cwt. = 23 cwt., $\therefore$ 1 cwt. cost £23 ÷ 23 = £1. Hence what must £1 be sold for if £100 is sold for £105?

$$\text{Selling price of } \overset{\pounds}{100} = \overset{\pounds}{105}$$

$$\text{,,} \qquad 1 = \frac{105}{100}$$

$$\pounds\frac{105}{100} = \pounds 1 \text{ 1s.} \quad \textit{Ans.}$$

(Ex. 107).—Eggs are bought at $7\frac{1}{2}$d. per dozen, and sold at 16 for a shilling. What is the gain per cent?

This sum may be worked out so as to give *three* answers. As before stated (par. **44**, p. 72), the *gain on outlay* is the correct answer, unless some other result is asked for. All three results will be shown. The example is most instructive. The last answer is the correct one.

(*a*) Gain per cent, in eggs, on outlay.

$$\text{Eggs bought for } \overset{\text{d.}}{7\tfrac{1}{2}} = \overset{\text{Eggs.}}{12}$$

$$\text{,,} \qquad 1 = \frac{12}{7\frac{1}{2}}$$

$$\text{,,} \qquad 12 = \frac{12 \times 12}{7\frac{1}{2}}$$

$$\frac{12 \times 12}{7\frac{1}{2}} = \frac{\overset{4}{\cancel{12}} \times 12 \times 2}{\underset{5}{\cancel{15}}} = \frac{96}{5} = 19\tfrac{1}{5} \text{ eggs bought for 1s.}$$

Since $19\frac{1}{5}$ eggs are bought, and 16 are sold for 1s.

Gain on 1s. outlay $= 19\frac{1}{5} - 16 = 3\frac{1}{5}$ eggs.

	d.		Eggs.
Gain on outlay of	12	$=$	$3\frac{1}{5}$
„	1	$=$	$\dfrac{3\frac{1}{5}}{12}$
„	100	$=$	$\dfrac{3\frac{1}{5}\times 100}{12}$

$$\frac{3\frac{1}{5}\times 100}{12}=\frac{\overset{4}{\cancel{16}}\times\overset{20}{\cancel{100}}}{\cancel{5}\times\underset{3}{\cancel{12}}}=\frac{80}{3}=26\tfrac{2}{3}\text{ \% gain in eggs on outlay.}\quad \textit{Ans.}$$

(*b*) Gain per cent on 12 eggs.

	Eggs.		d.
Selling price of	16	$=$	12
„	1	$=$	$\dfrac{12}{16}$
„	12	$=$	$\dfrac{12\times 12}{16}$

$$\frac{\overset{3}{\cancel{12}}\times\overset{3}{\cancel{12}}}{\underset{4}{\cancel{16}}}=9\text{d.}$$

Since 12 eggs sell for 9d., and cost only $7\frac{1}{2}$d., gain on 12 eggs $=1\frac{1}{2}$d.

	Eggs.		d.
Gain on	12	$=$	$1\frac{1}{2}$
„	1	$=$	$\dfrac{1\frac{1}{2}}{12}$
„	100	$=$	$\dfrac{1\frac{1}{2}\times 100}{12}$

$$\frac{1\frac{1}{2}\times 100}{12}=\frac{\cancel{3}\times\overset{25}{\cancel{100}}}{2\times\underset{4}{\cancel{12}}}=\frac{25}{2}=12\tfrac{1}{2}\text{ \% gain on 12 eggs.}\quad \textit{Ans.}$$

(*c*) Gain on outlay in money (par. **44**, p. 72).

Since 12 eggs sell for 9d., and cost $7\frac{1}{2}$d., gain = 9d. − $7\frac{1}{2}$d. = $1\frac{1}{2}$d.

$$\begin{array}{llr} & & \text{d.} \quad \text{d.} \\ \text{Gain on outlay of} & & 7\tfrac{1}{2} = 1\tfrac{1}{2} \\ \quad " \quad\quad " & & 1 = \dfrac{1\frac{1}{2}}{7\frac{1}{2}} \\ \quad " \quad\quad " & & 100 = \dfrac{1\frac{1}{2} \times 100}{7\frac{1}{2}} \end{array}$$

$$\frac{1\frac{1}{2} \times 100}{7\frac{1}{2}} = \frac{\frac{3}{\cancel{2}} \times 100 \times \cancel{2}}{\cancel{2} \times \cancel{15}_{5}} = \frac{100}{5} = 20\ \%\text{ gain on outlay.}$$

This is the correct Ans.

(Ex. 108.)—A woman buys a certain number of apples at 3 a penny, and a similar number at 2 a penny. She then mixes them and sells the whole at 5 for twopence. How much does she gain or lose per cent?

The least common multiple of 3, 2, and 5 is 30. Suppose, therefore, she buys 30 apples of each sort (*i.e.*, 60 in all), she pays for them 10d. + 15d. = 25d.

$$\begin{array}{lccl} & \text{Apples.} & & \text{Pence.} \\ \text{Selling price of} & 5 & = & 2 \\ \quad " & 1 & = & \tfrac{2}{5} \\ \quad " & 60 & = & \dfrac{2 \times 60}{5} = 24\text{d.} \end{array}$$

$$\begin{array}{lccl} & \text{Apples.} & & \text{Penny.} \\ \text{Loss on outlay of} & 25 & = & 1 \\ \quad " & 1 & = & \tfrac{1}{25} \\ \quad " & 100 & = & \dfrac{1 \times 100}{25} = 4\text{d. loss on 100d outlay.} \end{array}$$

$\therefore$ loss = 4%. *Ans.*

(Ex. 109.)—In a village school of 153 children, 125 passed in reading, writing, and arithmetic. What percentage passed in all three subjects?

$$\begin{array}{lr} & \text{Children.} \\ \text{Passes in } 153 = & 125 \\ \quad " \quad\quad 1 = & \dfrac{125}{153} \\ \quad " \quad\quad 100 = & \dfrac{125 \times 100}{153} \end{array}$$

$$\frac{125 \times 100}{153} = \frac{12500}{153} = 81\tfrac{107}{153}\%. \quad \textit{Ans.}$$

(Ex. 110.)—A maltster malts 7,500 bushels of barley, which in the process increases $12\frac{1}{2}$ per cent; how many bushels of malt has he?

100 bushels increase to $100+12\frac{1}{2}=112\frac{1}{2}$.

$$\begin{array}{lrl} & \text{Bushels.} & \text{Bushels.} \\ \text{Quantity from} & 100 & =112\frac{1}{2} \\ \quad " & 1 & =\dfrac{112\frac{1}{2}}{100} \\ \quad " & 7500 & =\dfrac{112\frac{1}{2}\times 7500}{100} \end{array}$$

$$\frac{112\frac{1}{2}\times 7500}{100}=\frac{225\times 75\cancel{00}}{2\times 1\cancel{00}}=\frac{16875}{2}=8437\tfrac{1}{2}\text{ bush. } \textit{Ans.}$$

(Ex. 111.)—In a school of 250 children 219 can write their names. What percentage is this of the whole?

$$\begin{array}{lrl} & \text{Children.} & \\ \text{Proportion in} & 250 & =219 \\ \quad " & 1 & =\dfrac{219}{250} \\ \quad " & 100 & =\dfrac{219\times 100}{260} \end{array}$$

$$\frac{219\times \overset{2}{\cancel{100}}}{\underset{5}{\cancel{250}}}=\frac{219\times 2}{5}=\frac{438}{5}=87\tfrac{3}{5}\ \%.\ \textit{Ans.}$$

45. Commission, Brokerage, Insurance.—These are all examples of Percentages, or of Interest, not reckoning time.

46. *Commission* is a percentage paid to an agent for the sale of property or goods.

47. *Brokerage* is a small percentage paid for transacting money concerns, especially the buying and selling of stock. It is a kind of commission. (See par. **54**, p. 91.)

48. *Insurance.*—Insurance is a percentage paid for securing property from loss by fire, &c. The money paid is called the *Premium*, and the agreement to pay the sum insured is called the *Policy of Insurance.**

(Ex. 112.)—What is the commission on £713 6s. 8d. at $2\frac{3}{4}$ per cent?

This simply means if £$2\frac{3}{4}$ is paid for selling £100 worth of goods, what must be paid for selling £713 6s. 8d. worth; which is the same as finding the simple interest on £713 6s. 8d. for one year at $2\frac{3}{4}$ per cent.

£713 6s. 8d. = £$713\frac{1}{3}$.

$$\begin{array}{lrl} & \pounds & \pounds \\ \text{Commission on} & 100 & = 2\frac{3}{4} \\ \quad ,, & 1 & = \dfrac{2\frac{3}{4}}{100} \\ \quad ,, & 713\frac{1}{3} & = \dfrac{2\frac{3}{4} \times 713\frac{1}{3}}{100} \end{array}$$

$$\pounds\frac{2\frac{3}{4} \times 713\frac{1}{3}}{100} = \pounds\frac{11 \times \overset{107}{\cancel{2140}}}{\underset{2}{\cancel{4}} \times 3 \times 10\cancel{0}} = \pounds\frac{11 \times 107}{6 \times 10} = \pounds\frac{1177}{60} = \pounds19\ 12\text{s. } 4\text{d. } \textit{Ans.}$$

(Ex. 113.)—What is the brokerage on £840 10s. at 2s. 6d. per cent?

This is another case of simple interest, without time.

£840 10s. = £$840\frac{1}{2}$. 2s. 6d. per cent = £$\frac{1}{8}$ paid on every £100.

$$\begin{array}{lrl} & \pounds & \pounds \\ \text{Brokerage on} & 100 & = \frac{1}{8} \\ \quad ,, & 1 & = \dfrac{\frac{1}{8}}{100} \\ \quad ,, & 840\frac{1}{2} & = \dfrac{\frac{1}{8} \times 840\frac{1}{2}}{100} \end{array}$$

$$\pounds\frac{\frac{1}{8} \times 840\frac{1}{2}}{100} = \pounds\frac{1 \times 1681}{8 \times 2 \times 100} = \pounds\frac{1681}{1600} = \pounds1\ 1\text{s. } 0\tfrac{3}{20}\text{d. } \textit{Ans.}$$

---

* When a percentage is paid to secure the payment of a sum of money at the death of a person, the policy is called a *Policy of Assurance.* In this, as in many other cases, the *Premium* is generally an annual one.

(Ex. 114.)—What is the brokerage on £852 10s. at $\frac{3}{8}$ per cent?

£852 10s. = £852$\frac{1}{2}$. $\frac{3}{8}\%$ = £$\frac{3}{8}$ paid on every £100.

$$\text{Brokerage on } \overset{\pounds}{100} = \overset{\pounds}{\tfrac{3}{8}}$$

$$\text{,,} \qquad 1 = \frac{\frac{3}{8}}{100}$$

$$\text{,,} \qquad 852\tfrac{1}{2} = \frac{\frac{3}{8} \times 852\frac{1}{2}}{100}$$

$$\pounds\frac{\frac{3}{8} \times 852\frac{1}{2}}{100} = \pounds\frac{3 \times \overset{341}{\cancel{1705}}}{8 \times 2 \times \underset{20}{\cancel{100}}} = \pounds\frac{1023}{320} = \pounds 3 \text{ 3s. } 11\tfrac{1}{4}\text{d.} \quad \textit{Ans.}$$

(Ex. 115.)—What is the cost of insuring a vessel and cargo worth £2,225 at $3\frac{1}{4}$ per cent?

In other words, find the simple interest on £2,225 at $3\frac{1}{4}$ per cent.

$$\text{Cost of insuring } \overset{\pounds}{100} = \overset{\pounds}{3\tfrac{1}{4}}$$

$$\text{,,} \qquad 1 = \frac{3\frac{1}{4}}{100}$$

$$\text{,,} \qquad 2225 = \frac{3\frac{1}{4} \times 2225}{100}$$

$$\pounds\frac{3\frac{1}{4} \times 2225}{100} = \pounds\frac{13 \times \overset{89}{\cancel{2225}}}{4 \times \underset{4}{\cancel{100}}} = \pounds\frac{1157}{16} = \pounds 72 \text{ 6s. 3d.} \quad \textit{Ans.}$$

(Ex. 116.)—What sum should be insured at 4 per cent on goods worth £735, so that, in case of loss, the owner may secure both the value of the goods and the premium paid?

Here, if £100 were insured, it would cover the loss of goods worth £96 together with £4 paid as premium.

$$\begin{array}{lr} & \pounds \quad \pounds \\ \text{Amount of Insurance on goods worth} & 96 = 100 \\ \quad ,, \qquad ,, \qquad ,, & 1 = \frac{100}{96} \\ \quad ,, \qquad ,, \qquad ,, & 735 = \frac{100 \times 735}{96} \end{array}$$

$$\pounds\frac{\overset{25}{\cancel{100}} \times 735}{\underset{24}{\cancel{96}}} = \pounds\frac{25 \times 735}{24} = \pounds\frac{18375}{24} = \pounds 765\ 12s.\ 6d. \quad \textit{Ans.}$$

(Ex. 117.)—What is the premium to be paid on a policy of life assurance of £6,968 for 2 years at $4\frac{3}{4}$ per cent?

In other words, find the simple interest on £6,968 for 2 years at $4\frac{3}{4}$ per cent.

$$\begin{array}{llll} & \pounds & \text{Years.} & \pounds \\ \text{Premium paid on} & 100 & \text{for } 1 & = 4\frac{3}{4} \\ \quad ,, & 1 & ,,\ 1 & \\ \quad ,, & 6968 & ,,\ 1 & = \dfrac{4\frac{3}{4} \times 6968 \times 2}{100} \\ \quad ,, & 6968 & ,,\ 2 & \end{array}$$

$$\pounds\frac{4\frac{3}{4} \times 6968 \times 2}{100} = \pounds\frac{19 \times \overset{871}{\cancel{6968}} \times \cancel{2}}{\underset{2}{\cancel{4}} \times \underset{25}{\cancel{100}}} = \pounds\frac{19 \times 871}{25} = \pounds\frac{16549}{25}$$

$$= \pounds 661\ 19s.\ 2\tfrac{2}{5}d. \quad \textit{Ans.}$$

49. Discount and Present Worth.—*Discount* is an allowance made for the payment of a sum of money before it is legally due. Discount is of two kinds.

(1) *Trade Discount.*—This is Simple Interest, calculated at the given rate (frequently 5 per cent) and for the given time, and *deducted* from the sum of money to be paid. Thus—

Mr. B. £100 – £5, *i.e.*, £95, and the debt is cancelled.

Farmer A. owes Mr. B. £100 for cattle, and he has to pay the money in 12 months time. But he wishes to pay at once. In such a case it is usual for the seller to deduct from the money owing as much *interest* as that money would make in the time before it becomes due, and to accept as payment the sum owing *minus* this interest. £100 in 12 months at 5% would gain £5 as interest. Farmer A., therefore, pays

(2) *True Discount.*—This is a deduction from the principal that leaves a sum which, if put out to interest at the given rate, and for the given time, would equal the original principal; or *True Discount* is the interest on the *Present Worth* of a sum of money, calculated from the present time to the time when the sum would be legally payable.

£100 put out to interest at 5% would amount in 12 months to £105, therefore £100 is the sum which must be paid at the present time to discharge a debt of £105, legally due in 12 months time. Hence £100 is called the *Present Worth* of £105, and the difference between £105 and £100, or £5, is the *true discount* on £105.

50. Finding the *present worth* of a sum of money due at the end of a given time, and at a given rate per cent, is exactly the same as finding the principal when the time, rate, and amount are given (par. **36** (3), p. 58, and Ex. 86).

As questions on Present Worth and True Discount often appear very complicated and are the cause of much confusion and frequent mistakes in working, it is a good plan to treat every sum in the same way, and *always employ one form of statement.* The answer required is either the discount or the present worth of a sum of money. *Always find the Discount first.* If discount is asked for, nothing further has to be done; if present worth is wanted, the given sum of money, *minus* the discount, gives the answer.

(Ex. 118.)—Find the true discount on £2,750, due two years hence, at $4\frac{1}{2}$ per cent.

Interest on £100 for 2 years @ $4\frac{1}{2}\% = £100 + (4\frac{1}{2} \times 2) = £100 + £9 = £109$.

$\therefore$ Present Worth of $£109 = £100$, and true discount on $£109 = £109 - £100 = £9$.

$$\text{True discount on } \overset{\pounds}{109} = \overset{\pounds}{9}$$

$$\quad\text{,,} \qquad 1 = \frac{9}{109}$$

$$\quad\text{,,} \qquad 2750 = \frac{9 \times 2750}{109}$$

$$\frac{\pounds 9 \times 2750}{109} = \pounds\frac{24750}{109} = \pounds 227 \text{ 1s. } 3\tfrac{45}{109}\text{d.} \quad \textit{Ans.}$$

(Ex. 119.)—What is the present value of £240, due two years hence at $3\frac{1}{2}$ per cent?

> First find the true discount, then £240 − true discount = present worth (or present value, as it is sometimes called).

Interest on £100 for 2 years at $3\frac{1}{2}$ per cent = £100 + ($3\frac{1}{2} \times 2$) = £100 + £7 = £107.

∴ Present worth of £107 = £100, and true discount on £107 = £107 − £100 = £7.

$$\text{True discount on } \overset{\pounds}{107} = \overset{\pounds}{7}$$

$$\quad\text{,,} \qquad 1 = \frac{7}{107}$$

$$\quad\text{,,} \qquad 240 = \frac{7 \times 240}{107}$$

$$\frac{\pounds 7 \times 240}{107} = \pounds\frac{1680}{107} = \pounds 15 \text{ 14s. } 0\tfrac{24}{107}\text{d, true dis.}$$

	£	s.	d.
Principal	240	0	0
Discount	15	14	$0\frac{24}{107}$
Present worth	224	5	$11\frac{83}{107}$ *Ans.*

(Ex. 120.)—Find the difference between the simple interest and the true discount on £100 for five years at 5 per cent.

Simple interest on £100 for 5 years at 5 per cent $= £100 + (£5 \times 5) = £100 + £25 = £125$.

$\therefore$ Present worth of £125 = £100, and true discount on £125 = £25.

$$\text{True discount on } \overset{£}{125} = \overset{£}{25}$$

$$\text{,,} \quad 1 = \frac{25}{125}$$

$$\text{,,} \quad 100 = \frac{25 \times 100}{125}$$

$$£\frac{\overset{20}{\cancel{25}} \times \cancel{100}}{\underset{5}{\cancel{125}}} = £20.$$

	£	s.	d.	
Simple interest	25	0	0	
True discount	20	0	0	
Difference	£5	0	0	*Ans.*

(Ex. 121.)—If the present worth of £218 due two years hence is £200; what is the present worth of £1,000, due six years hence at the same rate?

First find the rate of interest on £100 for a year.

£218 − £200 = £18 interest for 2 years, $\therefore$ £18 ÷ 2 = £9 the interest on £200 for 1 year, and £9 ÷ 2 = £4½ the interest on £100 for 1 year. Interest on £100 for 6 years at 4½% = £100 + (£4½ × 6) = £100 + £27 = £127.

Present worth of £127 = £100, and true discount on £127 = £27.

$$\text{True discount on } \overset{£}{127} = \overset{£}{27}$$

$$\text{,,} \quad 1 = \frac{27}{127}$$

$$\text{,,} \quad 1000 = \frac{27 \times 1000}{127}$$

$$£\frac{27 \times 1000}{127} = £\frac{27000}{127} = £212 \text{ 11s. } 11\tfrac{79}{127}\text{d.}$$

	£	s.	d.	
Principal	1000	0	0	
Discount	212	11	$11\frac{79}{127}$	
Present worth	787	8	$0\frac{48}{127}$.	*Ans.*

(Ex. 122.)—A merchant allows $2\frac{1}{2}$ per cent discount off all bills for cash. When he allows a discount of 22/6, what is the amount of the bill?

Here we have to find what principal will give 22/6 as interest if £100 gives £$2\frac{1}{2}$.

$$22/6 = £1\tfrac{1}{8}$$

$$\text{Principal to give a discount of } \overset{£}{2\tfrac{1}{2}} = \overset{£}{100}$$

$$\text{,, \quad ,, \quad } 1 = \frac{100}{2\frac{1}{2}}$$

$$\text{,, \quad ,, \quad } 1\tfrac{1}{8} = \frac{100 \times 1\frac{1}{8}}{2\frac{1}{2}}$$

$$\frac{£100 \times 1\frac{1}{8}}{2\frac{1}{2}} = £\frac{\overset{\overset{5}{\cancel{20}}}{\cancel{100}} \times 9 \times \cancel{2}}{\cancel{5} \times \underset{\cancel{4}}{\cancel{8}}} = £5 \times 9 = £45. \quad \textit{Ans.}$$

(Ex. 123.)—At 4 per cent per annum, what is the true discount on a bill of £100 due 3 months hence? If a discount of £1 were allowed, what would be the rate of interest?

$$\text{Interest on £100 for 3 months at } 4\% = £1$$

$$\text{True discount on £101 \quad ,, } = £1$$

$$\text{,, \quad 1 \quad ,, } = \frac{1}{101}$$

$$\text{,, \quad 100 \quad ,, } = \frac{1 \times 100}{101}$$

$$\frac{1 \times 100}{101} = \frac{100}{101} = 19s.\ 9\tfrac{63}{101}d. \quad \textit{Ans.}$$

Again, since discount is the interest on the present worth, £99 is the present worth of £100 due 3 months hence at 4%.

$$\text{Interest on £99 for 3 months} = £1$$

$$\left.\begin{array}{llll} \text{,,} & 1 & \text{,, } 3 & \text{,,} \\ \text{,,} & 1 & \text{,, } 1 & \text{,,} \\ \text{,,} & 100 & \text{,, } 1 & \text{,,} \\ \text{,,} & 100 & \text{,, } 12 & \text{,,} \end{array}\right\} = \frac{£1 \times 100 \times 12}{99 \times 3}$$

$$£\frac{1 \times 100 \times \overset{4}{\cancel{12}}}{99 \times \cancel{3}} = £\frac{400}{99} = £4\tfrac{4}{99}. \quad \textit{Ans.}$$

(Ex. 124.)—The discount on £226 2s. 8d. due at the end of 18 months is £12 16s. 0d. Find the rate of interest.

$$\pounds 12\ 16s. = \pounds 12\tfrac{4}{5}. \qquad \pounds 213\ 6s.\ 8d. = \pounds 213\tfrac{1}{3}.$$

Since discount is the interest on the present worth ∴ £12⅘ is the interest on £226 2s. 8d. − £12 16s. 0d. or £213 6s. 8d. That is, £213 6s. 8d. would, at the given rate per cent, gain £12⅘ as interest in 18 months.

Interest on £213⅓ for 18 months $= \pounds 12\tfrac{4}{5}$

$$\text{"} \quad 1 \quad \text{"} \quad 18 \quad \text{"} \quad = \pounds\frac{12\tfrac{4}{5}}{213\tfrac{1}{3}}$$

$$\text{"} \quad 1 \quad \text{"} \quad 1 \quad \text{"} \quad = \pounds\frac{12\tfrac{4}{5}}{213\tfrac{1}{3} \times 18}$$

$$\text{"} \quad 100 \quad \text{"} \quad 1 \quad \text{"} \quad = \pounds\frac{12\tfrac{4}{5} \times 100}{213\tfrac{1}{3} \times 18}$$

$$\text{"} \quad 100 \quad \text{"} \quad 12 \quad \text{"} \quad = \pounds\frac{12\tfrac{4}{5} \times 100 \times 12}{213\tfrac{1}{3} \times 18}$$

$$\pounds\frac{12\tfrac{4}{5} \times 100 \times 12}{213\tfrac{1}{3} \times 18} = \pounds\frac{\cancel{64} \times \overset{\overset{2}{\cancel{10}}}{\cancel{100}} \times \overset{2}{\cancel{12}} \times \cancel{3}}{\cancel{5} \times \underset{\cancel{10}}{\cancel{640}} \times \underset{\cancel{6}}{\cancel{18}}} = \pounds 2 \times 2 = \pounds 4 \text{ per cent. } \textit{Ans.}$$

(Ex. 125.)—What is the commission on the sale of goods worth £1243 19s. at ½ per cent?

½% means £½ for every £100

$$\begin{array}{lrrl} & \pounds & s. & \pounds \\ \text{Commission on} & 100 & 0 & = \tfrac{1}{2} \\ \quad\text{"} & 1 & 0 & = \dfrac{\tfrac{1}{2}}{100} \\ \quad\text{"} & 1243 & 19 & = \dfrac{\tfrac{1}{2} \times 1243\ 19s.}{100} \end{array}$$

$$\frac{\pounds\tfrac{1}{2} \times \pounds 1243\ 19s.}{100} = \frac{1 \times \pounds 1243\ 19s.}{2 \times 100} = \frac{\pounds 1243\ 19s.}{200} = \pounds 6\ 4s.\ 4\tfrac{37}{50}d. \quad \textit{Ans.}$$

(Ex. 126.)—What is the brokerage on £3964 14s. at 1½ per cent?

$$\begin{array}{lrl} & \pounds \quad s. & \pounds \\ \text{Brokerage on } & 100 \quad 0 & = 1\tfrac{1}{2} \\ \quad " & 1 \quad 0 & = \dfrac{1\frac{1}{2}}{100} \\ \quad " & 3964 \quad 14 & = \dfrac{1\frac{1}{2} \times 3964\ 14s.}{100} \end{array}$$

$$\frac{\pounds 1\frac{1}{2} \times \pounds 3964\ 14s.}{100} = \frac{3 \times \pounds 3964\ 14s.}{2 \times 100} = \frac{3 \times \pounds 1982\ 7s.}{100} = \frac{\pounds 5947\ 1s.}{100}$$

$$= \pounds 59\ 9s.\ 4\tfrac{23}{25}d. \quad \textit{Ans.}$$

(Ex. 127.)—I employ an agent to sell a quantity of goods, and agree to give him $\frac{3}{8}$ per cent upon the sales. The goods having sold for £7648, how much am I to pay him?

$$\begin{array}{lrl} & \pounds & \pounds \\ \text{Commission on } & 100 & = \tfrac{3}{8} \\ \quad " & 1 & = \dfrac{\frac{3}{8}}{100} \\ \quad " & 7648 & = \dfrac{\frac{3}{8} \times 7648}{100} \end{array}$$

$$\frac{\pounds\frac{3}{8} \times 7648}{100} = \pounds\frac{3 \times \overset{\overset{239}{956}}{\cancel{7648}}}{\cancel{8} \times \underset{25}{\cancel{100}}} = \pounds\frac{3 \times 239}{25} = \pounds\frac{717}{25} = \pounds 28\ 13s.\ 7\tfrac{1}{5}d. \quad \textit{Ans.}$$

(Ex. 128.)—If the rate of insurance be £2 8s. 6d. per cent, for what sum is a person insured who pays £2 17s. 9d.?

Here we have to find the principal on which a premium of £2 17s. 9d. is paid when £2 8s. 6d. is paid on £100.

$$\pounds 2\ 8s.\ 6d. = 48\tfrac{1}{2}s. \quad \pounds 2\ 17s.\ 9d. = 57\tfrac{3}{4}s.$$

$$\begin{array}{lrl} & \text{Shil.} & \pounds \\ \text{Principal paying } & 48\tfrac{1}{2} & = 100 \\ \quad " & 1 & = \dfrac{100}{48\frac{1}{2}} \\ \quad " & 57\tfrac{3}{4} & = \dfrac{100 \times 57\frac{3}{4}}{48\frac{1}{2}} \end{array}$$

$$\frac{\pounds 100 \times 57\frac{3}{4}}{48\frac{1}{2}}\ \pounds = \frac{\overset{25}{\cancel{100}} \times 231 \times 2}{97 \times \cancel{4}} = \pounds\frac{11550}{97} = \pounds 119\ 1s.\ 5\tfrac{31}{97}d. \quad \textit{Ans.}$$

51. Stocks and Shares.—Arithmetical problems included under the heading of Stocks and Shares are confessedly difficult for young students to understand. To them "Stock" is an intangible thing; they are always liable to confound Stock and Money, and thus endless mistakes arise.

52. Stock may be defined as Money borrowed by our own or other Governments, at so much per cent., to defray the expenses of the nation.

The money owing by England is called the *Funds*, or the *National Debt*. At the present time (1883) it amounts to about 762 millions. Interest is paid on the greater part of this sum at the rate of 3 per cent. The Government pledges itself to pay the interest regularly (every half-year), but reserves to itself the right of paying off the principal at any future time. (The interest and management of this immense sum of money cost, in 1882, about 29½ millions.*) Persons having a claim on the Government for interest are called *Fundholders*, and they are said to hold so much Stock, as £3,000 worth, £2,500 worth, &c. Though they can only demand from the Government that the *Dividends*—that is, the interest on the amount of stock they hold—be regularly paid, they can always turn their stock into money by selling their claim to another person. Stock sold to another person is said to be *Transferred;* the person selling is said to *Sell out*, and the person buying to *Invest*.

53. For convenience, Stock is divided into portions of £100 each, often called *cents*. The price at which these cents can be bought or sold varies slightly. When the price of £100 stock is £100 money, stock is said to be *at par*, when it is more than £100 money it is said to be *at a premium*, and when less than £100 money it is said to be *at a discount*.

54. *Stockbrokers* are agents who buy and sell stock for other persons. They are paid for their trouble by a

* For an excellent account of the National Debt, its rise, amount, and cost, see Whitaker's Almanack every year. Children should thoroughly understand all this, and that *above one-third* of all the taxes (not rates) raised every year goes to pay the expenditure on the National Debt.

commission or percentage, called *Brokerage.* (See par. **47**, p. 81.) This charge is generally £$\frac{1}{8}$ per cent on the stock bought and sold, so that when a person buys stock he pays £$\frac{1}{8}$ more for every cent than the market price, and when he sells out he gets £$\frac{1}{8}$ less than the market price for each cent.

Thus—If £100 stock is selling for £$94\frac{1}{8}$, the broker charges £$94\frac{1}{8} - \frac{1}{8}$, and the seller therefore only receives £94 for his £100 stock; but the person buying pays the broker £$94\frac{1}{8} + \frac{1}{8}$, or £$94\frac{1}{4}$ for the same amount of stock.

55. Shares form the capital of trading companies. This capital is divided into portions called shares, which people subscribe for, and according to the profits made these shares bear interest.

56. All questions in Stocks and Shares, and buying and selling money, are problems in Rule of Three, chiefly simple. The great thing to remember is that *Stock* is not *Cash* (money), *i.e.*, £100 stock does not mean, except very rarely, £100 money.

(Ex. 129.)—How much must be given for £1750 stock in the Three-and-a-Half per Cents, when the price is $96\frac{1}{4}$?

Here £100 stock, or one cent, costs £$96\frac{1}{4}$ money, what, therefore, is the cost of £1750 stock. The answer is evidently less than £1750, and will be *money.* The $3\frac{1}{2}$ per cent is not required.

$$\begin{array}{lll} & \text{£} & \text{£} \\ \text{Cost of} & 100 \text{ stock} & = 96\frac{1}{4} \text{ (money)} \\ \quad ,, & 1 \quad ,, & = \dfrac{96\frac{1}{4}}{100} \\ \quad ,, & 1750 \quad ,, & = \dfrac{96\frac{1}{4}\times 1750}{100} \end{array}$$

$$\frac{£96\frac{1}{4}\times 1750}{100} = £\frac{\overset{77}{\cancel{385}} \times \overset{350}{\cancel{1750}}}{4\times \underset{\underset{4}{\cancel{20}}}{\cancel{100}}} = £\frac{26950}{16} = £1684 \text{ 7s. 6d. } \textit{Ans.}$$

This question might have been asked thus: What is the value of £1750 stock in the Three-and-a-Half per Cents at $96\frac{1}{4}$?

(Ex. 130.)—How much stock in the Three per Cent Consols at $95\frac{1}{8}$ can I buy for £951 5s.?

The answer required is the quantity of stock that can be bought for £$951\frac{1}{4}$ money. The "Three per Cent" is not required.

$$\text{Stock purchased for } \overset{\pounds}{95\tfrac{1}{8}} = \overset{\pounds}{100} \text{ (stock)}$$

$$\text{,, \quad ,, \quad } 1 = \frac{100}{95\frac{1}{8}}$$

$$\text{,, \quad ,, \quad } 951\tfrac{1}{4} = \frac{100 \times 951\frac{1}{4}}{95\frac{1}{8}}$$

$$\frac{\pounds 100 \times 951\frac{1}{4}}{95\frac{1}{8}} = \pounds\frac{100 \times \overset{5}{\cancel{3805}} \times \overset{2}{\cancel{8}}}{\cancel{761} \times \cancel{4}} = \pounds 1000 \text{ stock. } \textit{Ans.}$$

(Ex. 131.)—What income is derived from £3550 stock in the Three per Cents at 95?

This is simply finding the interest on £3550 at 3 per cent. The £3550 being stock (not money) the 95 does not enter into the question, as every cent, irrespective of what it cost, produces £3 interest.

$$\text{Income from } \overset{\pounds}{100} \text{ stock} = \overset{\pounds}{3} \text{ (money)}$$

$$\text{,, \quad } 1 \text{ ,, } = \frac{3}{100}$$

$$\text{,, \quad } 3550 \text{ ,, } = \frac{3 \times 3550}{100}$$

$$\frac{\pounds 3 \times \overset{71}{\cancel{3550}}}{\underset{2}{\cancel{100}}} = \pounds\frac{213}{2} = \pounds 106 \text{ 10s. } \textit{Ans.}$$

(Ex. 132.)—A man invests £1274 in the Three-and-a-Half per Cents at 91. What income does he derive therefrom?

For every £91 (money) spent in the purchase of stock. £$3\frac{1}{2}$ is gained as interest; how much interest is gained by investing £1274 money?

$$\text{Income from } \overset{£}{91} \text{ money} = \overset{£}{3\tfrac{1}{2}} \text{ (money)}$$

$$\text{,,} \quad 1 \quad \text{,,} = \frac{3\frac{1}{2}}{91}$$

$$1274 \quad \text{,,} = \frac{3\frac{1}{2} \times 1274}{91}$$

$$\frac{£3\frac{1}{2} \times 1274}{91} = £\frac{7 \times \overset{\overset{7}{\cancel{14}}}{\cancel{1274}}}{\cancel{2} \times \cancel{91}} = £7 \times 7 = £49. \quad \textit{Ans}$$

(Ex. 133.)—What income is derived from investing £1227 in the Three-and-a-Half per Cents at $102\frac{1}{8}$, brokerage $\frac{1}{8}$?

As the brokerage has to be added to the price (par. **54**, p. 91), each cent costs £$102\frac{1}{8} + \frac{1}{8} =$ £$102\frac{1}{4}$ money. For every £$102\frac{1}{4}$ money invested, £$3\frac{1}{2}$ is gained as interest, ∴ how much is gained on £1227 money?

$$\text{Income from investing } \overset{£}{102\tfrac{1}{4}} \text{ money} = \overset{£}{3\tfrac{1}{2}} \text{ (money)}$$

$$\text{,,} \quad \text{,,} \quad 1 \quad \text{,,} = \frac{3\frac{1}{2}}{102\frac{1}{4}}$$

$$\text{,,} \quad \text{,,} \quad 1227 \quad \text{,,} = \frac{3\frac{1}{2} \times 1227}{102\frac{1}{4}}$$

$$\frac{£3\frac{1}{2} \times 1227}{102\frac{1}{4}} = £\frac{7 \times \overset{3}{\cancel{1227}} \times \overset{2}{\cancel{4}}}{\cancel{2} \times \cancel{409}} = £7 \times 3 \times 2 = £42. \quad \textit{Ans.}$$

This sum might have been worked by first finding how much stock the £1227 money purchased, viz., £1200 (as in Ex. 130), and then finding the interest £1200 stock produced, if £100 stock produced £$3\frac{1}{2}$ (as in Ex. 131).

(Ex. 134.)—Jones invests £5520 in the Three-and-a-Quarter per Cents when they are at 92; Robinson invests £6790 in the Three per Cents when they are at 97. Find the difference in their incomes?

Note.—£5520 and £6790 are sums of money, and *not* quantity of stock.

(*a*) Jones.

$$\text{Income from } \overset{\pounds}{92} \text{ money} = \overset{\pounds}{3\tfrac{1}{4}}$$

$$\text{,,} \quad 1 \quad \text{,,} = \frac{3\frac{1}{4}}{92}$$

$$\text{,,} \quad 5520 \quad \text{,,} = \frac{3\frac{1}{4} \times 5520}{92}$$

$$\pounds\frac{3\frac{1}{4} \times \overset{60}{\cancel{5520}}}{\cancel{92}} = \pounds 3\tfrac{1}{4} \times 60 = \pounds 195.$$

(*b*) Robinson.

$$\text{Income from } \overset{\pounds}{97} \text{ money} = \overset{\pounds}{3}$$

$$\text{,,} \quad 1 \quad \text{,,} = \frac{3}{97}$$

$$\text{,,} \quad 6790 \quad \text{,,} = \frac{3 \times 6790}{97}$$

$$\pounds\frac{3 \times \overset{70}{\cancel{6790}}}{\cancel{97}} = \pounds 210.$$

$\therefore$ difference $= \pounds 210 - \pounds 195 = \pounds 15$. *Ans.*

(Ex. 135.)—If the Four per Cents are at 95, what rate per cent per annum do I get for my money?

In other words—If £95 money gives £4 interest, what will £100 give?

$$\text{Interest on } \overset{\pounds}{95} \text{ money} = \overset{\pounds}{4}$$

$$\text{,,} \quad 1 \quad \text{,,} = \frac{4}{95}$$

$$\text{,,} \quad 100 \quad \text{,,} = \frac{4 \times 100}{95}$$

$$\pounds\frac{4 \times \overset{20}{\cancel{100}}}{\underset{19}{\cancel{95}}} = \pounds\frac{80}{19} = \pounds 4\tfrac{4}{19} \text{ per cent.} \quad \textit{Ans.}$$

(Ex. 136.)—At what price must I purchase stock in the Three-and-a-Half per Cents, so as to get 4 per cent on my outlay?

In other words—If a principal of £100 gives £4 as interest, what sum of money will give £3½?

$$\begin{array}{lcl} & \pounds & \pounds \\ \text{Principal to give} & 4 \text{ interest} & = 100 \\ \quad \text{"} & 1 \quad \text{"} & = \dfrac{100}{4} \\ \quad \text{"} & 3\tfrac{1}{2} \quad \text{"} & = \dfrac{100 \times 3\tfrac{1}{2}}{4} \end{array}$$

$$\pounds\frac{100 \times 3\frac{1}{2}}{4} = \pounds\frac{\overset{25}{\cancel{100}} \times 7}{\cancel{4} \times 2} = \pounds\frac{175}{2} = \pounds 87\tfrac{1}{2}. \quad \textit{Ans.}$$

Ex. 137.)—If I invest £1,200 in the Four per Cents at 72, what is my half-yearly dividend?

Here £72 money produces a yearly dividend of £4, or a half-yearly dividend of £2; what dividend, therefore, will £1,200 money produce?

$$\begin{array}{lcl} & \pounds & \pounds \\ \text{Dividend on} & 72 \text{ money} & = 2 \\ \quad \text{"} & 1 \quad \text{"} & = \dfrac{2}{72} \\ \quad \text{"} & 1200 \quad \text{"} & = \dfrac{2 \times 1200}{72} \end{array}$$

$$\pounds\frac{\cancel{2} \times \overset{100}{\cancel{1200}}}{\underset{\underset{3}{\cancel{6}}}{\cancel{72}}} = \pounds\frac{100}{3} = \pounds 33 \text{ 6s. 8d.} \quad \textit{Ans.}$$

(Ex. 138.)—The Three-and-a-Half per Cents are at 99⅞; how much money must be invested in them to produce an income of £280?

Here a principal of £99⅞ gives an income of £3½; how much money, therefore, gives an income of £280?

$$\begin{array}{llr} & & \text{£} \quad\ \text{£} \\ \text{Principal to give an income of} & & 3\tfrac{1}{2} = 99\tfrac{7}{8} \\ \quad ,, & ,, & 1 = \dfrac{99\tfrac{7}{8}}{3\tfrac{1}{2}} \\ \quad ,, & ,, & 280 = \dfrac{99\tfrac{7}{8} \times 280}{3\tfrac{1}{2}} \end{array}$$

$$\frac{\text{£}99\tfrac{7}{8} \times 280}{3\tfrac{1}{2}} = \text{£}\frac{799 \times \overset{\overset{5}{\cancel{35}}}{\cancel{280}} \times 2}{\cancel{8} \times \cancel{7}} = \text{£}799 \times 10 = \text{£}7990 \ \textit{Ans.}$$

(Ex. 139.)—A thousand pounds is invested in the Four-and-a-Half per Cents at 95, and also in the Five per Cents. What price must the Five per Cents be at so as to produce the same income as the Four-and-a-Half per Cents?

In other words, if a principal of £95 gives £4½ interest, what principal will give £5 interest?

$$\begin{array}{lr} & \text{£} \quad \text{£} \\ \text{Principal to give} & 4\tfrac{1}{2} = 95 \\ \quad ,, & 1 = \dfrac{95}{4\tfrac{1}{2}} \\ \quad ,, & 5 = \dfrac{95 \times 5}{4\tfrac{1}{2}} \end{array}$$

$$\frac{\text{£}95 \times 5}{4\tfrac{1}{2}} = \text{£}\frac{95 \times 5 \times 2}{9}\ \text{£}\frac{950}{9} = \text{£}105\tfrac{5}{9} \ \textit{Ans.}$$

i.e. the Five per Cents must be selling at £105⅝.

(Ex. 140.)—Which is the best stock to invest in, the Three per Cents at 89½, or the Three-and-a-Half per Cents at 98½?

There are several ways in which this question may be worked. The following is, perhaps, the best. If £89½ invested in the Three per Cents produces £3, how much would be produced if the

same sum of money (£$89\frac{1}{2}$) were invested in the Three-and-a-Half per Cents at $98\frac{1}{2}$?

$$\begin{array}{rrl} & £ & £ \\ \text{Income from} & 98\tfrac{1}{2} & = 3\tfrac{1}{2} \\ ,, & 1 & = \dfrac{3\frac{1}{2}}{98\frac{1}{2}} \\ ,, & 89\tfrac{1}{2} & = \dfrac{3\frac{1}{2} \times 89\frac{1}{2}}{98\frac{1}{2}} \end{array}$$

$$£\frac{3\frac{1}{2} \times 89\frac{1}{2}}{98\frac{1}{2}} = £\frac{7 \times 179 \times \cancel{2}}{2 \times \cancel{2} \times 197} = £\frac{1253}{394} = £3\tfrac{71}{394}$$

In first case £$89\frac{1}{2}$ produces £3

,, second ,, £$3\frac{71}{394}$

∴ the Three-and-a-half per Cents is the best investment. *Ans.*

(Ex. 141.)—I sell £43,400 Stock out of the Three per Cents at 96 and buy Six per Cent Debenture Stock at 105. Find the difference in my income, and the gain or loss per cent?

There are five operations in this sum—

(*a*) Find income from £43,400 Stock in the Three per Cents.
(*b*) ,, amount realised from sale of £43,400 Stock at 96.
(*c*) ,, income in second case.
(*d*) ,, difference between incomes *a* and *c*.
(*e*) ,, gain or loss per cent on the income produced from £1 invested in the two cases.

$$\begin{array}{rrrl} & & £ & £ \\ (a) & \text{Income from} & 100 \text{ stock} & = 3 \\ ,, & & 1 \;\; ,, & = \dfrac{3}{100} \\ ,, & & 43400 \;\; ,, & = \dfrac{3 \times 43400}{100} \end{array}$$

$$\frac{£3 \times 434\cancel{00}}{1\cancel{00}} = £1302. \quad \text{First income.}$$

$$\text{(b) Sale price of 100 stock} \overset{\pounds}{} = \overset{\pounds}{96} \text{ (money)}$$

$$\text{,,} \quad 1 \quad \text{,,} \quad = \frac{96}{100}$$

$$\text{,,} \quad 43400 \quad \text{,,} \quad = \frac{96 \times 43400}{100}$$

$\pounds\frac{96 \times 434\cancel{00}}{1\cancel{00}} = \pounds41664.$ Amount invested in Six per Cent Debenture Stock.

$$\text{(c) Income from } \overset{\pounds}{105} \text{ money} = \overset{\pounds}{6}$$

$$\text{,,} \quad 1 \quad \text{,,} \quad = \frac{6}{105}$$

$$\text{,,} \quad 41664 \quad \text{,,} \quad = \frac{6 \times 41664}{105}$$

$\frac{6 \times 41664}{105} = \pounds\frac{249984}{105} = \pounds2380$ 16s. Second income.

(*d*) The second income is the greatest $\therefore$ difference =

£2380 16s. − £1302 = £1078 16s. gain. *Ans.*

$$\text{(e) Income in 1st case from investing } \overset{\pounds}{96} = \overset{\pounds}{3}$$

$$\text{,,} \quad \text{,,} \quad \text{,,} \quad 1 = \frac{3}{96} = \pounds\frac{1}{32}$$

$$\text{Income in 2nd case from investing } \overset{\pounds}{105} = \overset{\pounds}{6}$$

$$\text{,,} \quad \text{,,} \quad \text{,,} \quad 1 = \frac{6}{105} = \pounds\frac{2}{35}$$

$$\text{Difference in income on every £1 invested} = \pounds\frac{2}{35} - \pounds\frac{1}{32} = \pounds\frac{29}{1120}$$

$$\therefore \text{ difference per cent, or on £100} = \pounds\frac{29 \times 100}{1120} = \pounds\frac{2900}{1120} = \pounds2\tfrac{33}{56}\%$$

Ans.

(Ex. 142.)—I sell out £4,500 Five per Cent Stock at 112½, and invest the proceeds in Foreign Stock yielding 7 per cent. My income is increased by £168 15s.; find the price of the Foreign Stock.

Note that the £4,500 is not money, but stock. There are here five operations :—

(1) Money produced from sale of Five per Cent Stock.
(2) Income from this stock.
(3) Income from Foreign Stock.
(4) Amount of Foreign Stock.
(5) Price per £100 of Foreign Stock.

$$\text{(1) Selling price of } \overset{\pounds}{100} \text{ stock} = \overset{\pounds}{112\tfrac{1}{2}} \quad \text{(Money.)}$$

$$\text{"} \qquad 1 \quad \text{"} \quad = \frac{112\frac{1}{2}}{100}$$

$$\text{"} \qquad 4500 \quad \text{"} \quad = \frac{112\frac{1}{2} \times 4500}{100}$$

$$\frac{\pounds 112\frac{1}{2} \times 45\cancel{00}}{1\cancel{00}} = \pounds 5062\tfrac{1}{2}. \quad \text{(Money.)}$$

$$\text{(2) Income from } \overset{\pounds}{100} \text{ stock} = \overset{\pounds}{5} \quad \text{(Money.)}$$

$$\text{"} \qquad 1 \quad \text{"} \quad = \frac{5}{100}$$

$$\text{"} \qquad 4500 \quad \text{"} \quad = \frac{5 \times 4500}{100}$$

$$\frac{\pounds 5 \times 45\cancel{00}}{1\cancel{00}} = \pounds 225. \quad \text{(Money.)}$$

(3) Income from Foreign Stock = £225 + £168 15s. = £393 15s. = £393¾.

$$\text{(4) Amount of stock giving income of } \overset{\pounds}{7} = \overset{\pounds}{100} \quad \text{(Stock.)}$$

$$\text{"} \qquad \text{"} \qquad \text{"} \qquad 1 = \frac{100}{7}$$

$$\text{"} \qquad \text{"} \qquad \text{"} \qquad 393\tfrac{3}{4} = \frac{100 \times 393\frac{3}{4}}{7}$$

$$\frac{100 \times 393\frac{3}{4}}{7} = \frac{\pounds 100 \times \overset{225}{\cancel{1575}}}{\cancel{7} \times 4} = \frac{22500}{4} = \pounds 5625. \quad \text{(Stock.)}$$

$$(5)\ \text{Price paid for } \overset{\pounds}{5625} \text{ stock} = \overset{\pounds}{5062\tfrac{1}{2}} \text{ (Money).}$$

$$\text{,,} \quad 1 \quad \text{,,} \quad = \frac{5062\frac{1}{2}}{5625}$$

$$\text{,,} \quad 100 \quad \text{,,} \quad = \frac{5062\frac{1}{2} \times 100}{5625}$$

$$\frac{\pounds 5062\frac{1}{2} \times \overset{4}{\cancel{100}}}{\underset{225}{\cancel{5625}}} = \frac{\pounds 20250}{225} = \pounds 90. \quad \textit{Ans.}$$

(Ex. 143.)—What must be the price of Three per Cent Consols so that, by investing £32,850 my income may be £1,080 a year?

In other words, if an investment of £32850 (money) produces £1080, how much will produce £3?

$$\text{Money to produce } \overset{\pounds}{1080} = \overset{\pounds}{32850}$$

$$\text{,,} \quad 1 \quad = \frac{32850}{1080}$$

$$\text{,,} \quad 3 \quad = \frac{32850 \times 3}{1080}$$

$$\frac{3285\cancel{0} \times \cancel{3}}{\underset{36\cancel{0}}{\cancel{1080}}} = \frac{3285}{36} = \pounds 91\tfrac{1}{4}.$$

The Three per Cent Consols must be at £91¼. *Ans.*

(Ex. 144.)—What will be the clear annual income derived from investing £6050 in the Three per Cents at 90¾, after deducting an income tax of 4d. in the pound?

First find the gross income from investing £6050 (money), and then deduct the income tax.

$$\text{Income from } \overset{\pounds}{90\tfrac{3}{4}} = \overset{\pounds}{3}$$

$$\text{,,} \quad 1 = \frac{3}{90\frac{3}{4}}$$

$$\text{,,} \quad 6050 = \frac{3 \times 6050}{90\frac{3}{4}}$$

$$\frac{\pounds \cancel{3} \times 6050 \times 4}{\underset{121}{\cancel{363}}} = \pounds\frac{24200}{121} = \pounds 200.$$

Then £200 − (4d. × 200) = £200 − £3 6 8 = £196 13s. 4d. *Ans.*

(Ex. 145.)—How much must I invest in the Three-and-a-Half per Cents at 91, so as to have an annual income of £932, after deducting an income tax of 7d. in the £?

Here £91 will produce an income of £3½ minus the tax.

$$7\text{d. in the £ on £}3\tfrac{1}{2} = £\frac{7 \times 3\frac{1}{2}}{240} = £\frac{49}{480}$$

$$\text{Then £}3\tfrac{1}{2} - £\frac{49}{480} = £\frac{7}{2} - £\frac{49}{480} = £\frac{1680 - 49}{480} = £\frac{1631}{480}$$

$$\text{Sum to be invested to produce } \overset{£}{\frac{1631}{480}} = \overset{£}{91}$$

$$\text{,, ,, } 1 = \frac{91}{\frac{1631}{480}}$$

$$\text{,, ,, } 932 = \frac{91 \times 932}{\frac{1631}{480}}$$

$$\frac{£91 \times 932}{\frac{1631}{480}} = £\frac{\overset{13}{\cancel{91}} + \overset{4}{\cancel{932}} \times 480}{\underset{\cancel{233}}{\cancel{1631}}} = £24960. \quad \textit{Ans.}$$

(Ex. 146.)—What must be the selling price of Three per Cent Consols so that after deducting an income tax of 6d. in the pound it may yield 3½ per cent interest on the outlay?

£1 minus 6d. tax = 19½s. ∴ if for 19½s. we must receive £1 in order to cover the tax, what must be received instead of £3½ or 70s. in order to cover the tax?

$$\text{Amount received for } \overset{\text{Shil.}}{19\tfrac{1}{2}} = \overset{£}{1}$$

$$\text{,, } 1 = \frac{1}{19\frac{1}{2}}$$

$$\text{,, } 70 = \frac{1 \times 70}{19\frac{1}{2}}$$

$$\frac{£1 \times 70}{19\frac{1}{2}} = £\frac{70 \times 2}{39} = £\frac{140}{39} = £3\tfrac{23}{39} \left\{ \begin{array}{l} \text{the interest to be obtained on} \\ \text{every £100 stock in order to} \\ \text{cover the tax.} \end{array} \right.$$

The question now resolves itself thus: What must be the selling price of the Three per Cents so that $£3\frac{23}{39}$ interest may be yielded? Or, what principal will give £3 interest if $£3\frac{23}{39}$ is yielded by £100?

$$\text{Principal to yield } £3\tfrac{23}{39} \text{ or } \overset{£}{\frac{140}{39}} = \overset{£}{100}$$

$$\text{,,} \qquad 1 = \frac{100}{\frac{140}{39}}$$

$$\text{,,} \qquad 3 = \frac{100 \times 3}{\frac{140}{39}}$$

$$£\frac{100 \times 3}{\frac{140}{39}} = £\frac{\overset{5}{\cancel{100}} \times 3 \times 39}{\underset{7}{\cancel{140}}} = £\frac{585}{7} = £83\tfrac{4}{7}$$

$$= £83\ 11\ 5\tfrac{1}{7}. \quad \textit{Ans.}$$

(Ex. 147.)—I hold £20,000 stock in the Three per Cents at £10 discount and transfer it to the Four per Cents at £15 premium; how much stock do I hold in the latter case, and find the alteration in my income, charging brokerage $\frac{1}{8}$ per cent on each transaction?

At £10 discount the price of £100 stock $= £100 - £10 = £90$.

At £15 premium the price of £100 stock $= £100 + £15 = £115$.

Selling out, in the first case, each cent is sold for $90 - \frac{1}{8} = 89\frac{7}{8}$.

Buying, in the second case, each cent is bought for $115 + \frac{1}{8} = 115\frac{1}{8}$.

$$(a) \text{ Amount of stock, each cent costing } \overset{£}{89\tfrac{7}{8}} = \overset{£}{20000}$$

$$\text{,,} \qquad \text{,,} \qquad 1 = 20000 \times 89\tfrac{7}{8}$$

$$\text{,,} \qquad \text{,,} \qquad 115\tfrac{1}{8} = \frac{20000 \times 89\frac{7}{8}}{115\frac{1}{8}}$$

$$£\frac{20000 \times 89\frac{7}{8}}{115\frac{1}{8}} = £\frac{20000 \times 719 \times \cancel{8}}{921 \times \cancel{8}} = £\frac{14380000}{921} = £15613\tfrac{427}{921} \text{ stock.}$$

Note the reasoning in the second step. If each cent costs only £1, more cents can be bought than if they cost $£89\frac{7}{8}$ each.

(*b*) Income in first case—

$$\text{Income from } \overset{\pounds}{100} \text{ stock} = \overset{\pounds}{3}$$

$$\text{,,} \quad 1 \quad \text{,,} \quad = \frac{3}{100}$$

$$\text{,,} \quad 20000 \quad \text{,,} \quad = \frac{3 \times 20000}{100}$$

$$\frac{\pounds 3 \times 200\cancel{00}}{1\cancel{00}} = \pounds 600.$$

(*c*) Income in second case—

$$\text{Income from} \quad \overset{\pounds}{100} \text{ stock} = \overset{\pounds}{4}$$

$$\text{,,} \quad 1 \quad \text{,,} \quad = \frac{4}{100}$$

$$\text{,,} \quad \pounds 15613\tfrac{567}{921} \text{ or } \frac{14380000}{921} \quad \text{,,} \quad = \frac{4 \times 14380000}{921 \times 100}$$

$$\frac{\pounds 4 \times 14380000}{921 \times 1\cancel{00}} = \pounds\frac{575200}{921} = \pounds 624 \text{ 10s. } 9\tfrac{77}{307}\text{d.}$$

(*d*) Alteration of income $= \pounds 624$ 10s. $9\frac{77}{307}$d. $- \pounds 600$
$= \pounds 24$ 10s. $9\frac{77}{307}$d. *Ans.*

(Ex. 148.)—A person invests £10,000 in Three per Cents at 75, and when they rise to 78 he sells out and invests the produce in bank shares at £208 each, which pay a dividend of £8 per share. Show that his income is not altered.

Income from every £75 invested = £3

Sum produced by every £75 sold out = £78

$$\text{Income from investing } \overset{\pounds}{208} = \overset{\pounds}{8}$$

$$\text{,,} \quad \text{,,} \quad 1 = \frac{8}{208}$$

$$\text{,,} \quad \text{,,} \quad 78 = \frac{8 \times 78}{208}$$

$$\pounds\frac{\cancel{8} \times 78}{\underset{26}{\cancel{208}}} = \pounds\frac{78}{26} = \pounds 3. \quad \textit{Ans.}$$

(Ex. 149.)—By purchasing railway shares at $13\frac{1}{2}$ per cent discount and selling them at $5\frac{1}{4}$ per cent premium, I gain £300. What was the original sum I expended?

$13\frac{1}{2}\% + 5\frac{1}{4}\% = 18\frac{3}{4}$, the total percentage gained.

$£100 - £13\frac{1}{2} = £86\frac{1}{2} =$ price of shares.

$$\begin{array}{lll} \text{Money expended on share giving a gain of} & \overset{£}{18\frac{3}{4}} & = \overset{£}{86\frac{1}{2}} \\ \quad " \qquad\qquad\qquad " & 1 & = \dfrac{86\frac{1}{2}}{18\frac{3}{4}} \\ \quad " \qquad\qquad\qquad " & 300 & = \dfrac{86\frac{1}{2} \times 300}{18\frac{3}{4}} \end{array}$$

$$\frac{86\frac{1}{2} \times 300}{18\frac{3}{4}} = \frac{173 \times \overset{4}{\cancel{300}} \times \overset{2}{\cancel{4}}}{\cancel{2} \times \cancel{75}} = £1384. \quad \textit{Ans.}$$

(Ex. 150.)—The Three per Cent Consols are paid on 5th January. What rate per cent. is obtained by buying consols on 23rd April at $93\frac{5}{8}$, paying broker's charge?

First find the dividend accruing between January 5th and April 23, or 108 days, which, deducted from the $93\frac{5}{8}$, will give the *net price* of the Consols; for this dividend, though not paid until 5th July [par. **52**, p. 91], is due to the purchaser, and therefore in the end reduces, by so much, the amount he pays for every cent. From the net price of Consols then find the rate per cent.

$$\begin{array}{llll} (a) \text{ Interest on } \overset{£}{100} \text{ for} & \overset{\text{Days}}{365} & = \overset{£}{3} & \\ \quad " \qquad " & 1 & = \dfrac{3}{365} & \\ \quad " \qquad " & 108 & = \dfrac{3 \times 108}{365} = \dfrac{324}{365} & = 17/9 + \end{array}$$

(b) Price of consols + brokerage of $\frac{1}{8} = £93\frac{5}{8} + \frac{1}{8} = £93\frac{6}{8} =$

	£	s.	d.
	£93	15	0
Less dividend for 108 days		17	9
Net price	£92	17	3 $= £92\frac{69}{80}$

$$\begin{array}{lll} (c) \text{ Interest on } \overset{£}{92\frac{69}{80}} & = \overset{£}{3} \\ \quad " \qquad 1 & = \dfrac{3}{92\frac{69}{80}} \\ \quad " \qquad 100 & = \dfrac{3 \times 100}{92\frac{69}{80}} \end{array}$$

$$\frac{£3 \times 100}{92\frac{69}{80}} = £\frac{3 \times 100 \times 80}{7429} = £\frac{24000}{7429} = £3 \ 4\text{s. } 7\text{d.} + \textit{Ans.}$$

57. Notes of a Lesson on Simple

MATTER.

I.—Solving the Problem. (Ex. 6, p. 17.)

If 4 men earn 16 shillings, how much will 13 men earn?

(*i.*)—Divide the problem into its two parts.

(*ii.*)—Consider which of the terms in the statement the answer has to be like, viz., shillings, and write down the statement in such a way that this term is the last in the line.

(*iii.*)—Reduce the first term of the arranged statement to unity, forming a fraction representing its value.

(*iv.*)—Introduce the term in the demand in place of unity, still keeping a fractional form.

(*v.*)—Reduce the fraction—

(*a*) by multiplication and division.

(*b*) or by cancelling.

Proportion by Method of Unity.

METHOD.

(*a*) *Statement*—4 men earn 16s.

(*b*) *Demand*—how much will 13 men earn?

Men. Shill.
Wages earned by 4 = 16

„ „ $1=\frac{16}{4}$ { One man will earn 4 times less than 4 men, ∴ 16 divided by 4 will be the wages of one man.

„ „ $13=\frac{16\times13}{4}$ { 13 men will earn 13 times as much as 1 man, ∴ the wages of 1 man multiplied by 13 will give the required answer.

$\frac{16\times13}{4}=\frac{208}{4}=52/-=£2\ 12s.$ *Ans.*

$\frac{\overset{4}{\cancel{16}}\times13}{\cancel{4}}=52/-=£2\ 12s.$ *Ans.*

Note.—Any of the exercises, but especially those with explanatory notes, can be easily formed into **Notes of Lessons** on this plan.

APPENDIX I.

SIMPLE PROPORTION.*

(Ex. 151.)—A servant's wages for 3 months is £3 2s. 6d.; how much is that for $2\frac{1}{2}$ years?

$2\frac{1}{2}$ years = 30 months.

$$\text{Wages for } \overset{\text{Months.}}{3} = \pounds 3\ 2\text{s.}\ 6\text{d.}$$

$$\text{,,}\quad 1 = \frac{\pounds 3\ 2\text{s.}\ 6\text{d.}}{3}$$

$$\text{,,}\quad 30 = \frac{\pounds 3\ 2\text{s.}\ 6\text{d.} \times 30}{3}$$

$$\frac{\pounds 3\ 2\text{s.}\ 6\text{d.} \times \overset{10}{\cancel{30}}}{\cancel{3}} = \pounds 31\ 5\text{s.}\quad \textit{Ans.}$$

(Ex. 152.)—How many stone of beef can I buy for £5 7s. 4d., if I pay 7/8 for 8lb.?

£5 7s. 4d. = 1288d. 7/8 = 92d.

$$\text{Pounds for } \overset{\text{d.}}{92} = \overset{\text{lbs.}}{8}$$

$$\text{,,}\quad 1 = \frac{8}{92}$$

$$\text{,,}\quad 1288 = \frac{8 \times 1288}{92}$$

$$\frac{8 \times \overset{14}{\cancel{1288}}}{\cancel{92}} = 112\text{lb.} = 8 \text{ stone.}\quad \textit{Ans.}$$

---

* This appendix contains all the questions in John Heywood's Series of Home Lesson Books, Standard V. and VI., not worked out in the body of the work, to which the Method of Unity is applicable. References are made in the answer books to the number prefixed to these exercises.

(Ex. 153.)—If 13lbs. of sugar cost 4/4, how many lbs. can be bought for £3 12s. 4d.?

4/4 = 52d. £3 12s. 4d. = 868d.

$$\begin{array}{lrcl} & \text{d.} & & \text{lbs.} \\ \text{Weight for} & 52 & = & 13 \\ \quad " & 1 & = & \frac{13}{52} \\ \quad " & 868 & = & \frac{13 \times 868}{52} \end{array}$$

$$\frac{\cancel{13} \times 868}{\underset{4}{\cancel{52}}} = \frac{868}{4} = 217\text{lbs.} \quad \textit{Ans.}$$

(Ex. 154.)—I spend 18 guineas in 36 days. At the end of the year I find I have saved 100 guineas. What is my income?

$$\begin{array}{lrcl} & \text{Days.} & & \text{Guineas.} \\ \text{Amount spent in} & 36 & = & 18 \\ \quad " & 1 & = & \frac{18}{36} \\ \quad " & 365 & = & \frac{18 \times 365}{36} \end{array}$$

$$\frac{\cancel{18} \times 365}{\underset{2}{\cancel{36}}} = \frac{365}{2} = 182\tfrac{1}{2} \text{ guineas.} \quad \textit{Ans.}$$
$$= 182\text{gu. } 10\text{s. } 6\text{d.} \quad "$$
$$= £191 \ 12\text{s. } 6\text{d.} \quad "$$

(Ex. 155.)—If the rent of a house be £10 a year, how much is that for 10 weeks?

$$\begin{array}{lrcl} & \text{Weeks.} & & £ \\ \text{Rent for} & 52 & = & 10 \\ \quad " & 1 & = & \frac{10}{52} \\ \quad " & 10 & = & \frac{10 \times 10}{52} \end{array}$$

$$£\frac{10 \times \overset{5}{\cancel{10}}}{\underset{26}{\cancel{52}}} = £\frac{50}{26} = £1 \ 18\text{s. } 5\tfrac{1}{2}\text{d.} + \tfrac{2}{13}\text{f., (or } 5\tfrac{7}{13}\text{d.)} \quad \textit{Ans.}$$

(Ex. 156.)—If 6 yards of cloth cost £1 1s., how much must be given for 10 yards?

$$\pounds 1\ 1\text{s.} = 21\text{s.}$$

$$\begin{array}{lccc} & \text{Yards.} & & \text{Shillings.} \\ \text{Price for} & 6 & = & 21 \\ \quad ,, & 1 & = & \dfrac{21}{6} \\ \quad ,, & 10 & = & \dfrac{21 \times 10}{6} \end{array}$$

$$\frac{21 \times \overset{5}{\cancel{10}}}{\underset{3}{\cancel{6}}}\text{s.} = \frac{105}{3}\text{s.} = 35\text{s.} = \pounds 1\ 15\text{s.} \quad \textit{Ans.}$$

(Ex. 157.)—If 2 loads of hay last 6 horses for a week, how many loads will 24 horses eat in the same time?

$$\begin{array}{lccc} & \text{Horses.} & & \text{Loads.} \\ \text{Quantity for} & 6 & = & 2 \\ \quad ,, & 1 & = & \dfrac{2}{6} \\ \quad ,, & 24 & = & \dfrac{2 \times 24}{6} \end{array}$$

$$\frac{2 \times \overset{4}{\cancel{24}}}{\cancel{6}} = 8 \text{ loads.} \quad \textit{Ans.}$$

(Ex. 158.)—How much must I pay for 20 loaves of bread, if 6 loaves cost me 4s. $1\frac{1}{2}$d.?

$$\begin{array}{lcl} & \text{Loaves.} & \\ \text{Price for} & 6 & = 4/1\tfrac{1}{2} \\ \quad ,, & 1 & = \dfrac{4/1\frac{1}{2}}{6} \\ \quad ,, & 20 & = \dfrac{4/1\frac{1}{2} \times 20}{6} \end{array}$$

$$\frac{4/1\frac{1}{2} \times \overset{10}{\cancel{20}}}{\underset{3}{\cancel{6}}} = \frac{41/3}{3} = 13\text{s. } 9\text{d.} \quad \textit{Ans.}$$

(Ex. 159.)—If $2\frac{1}{2}$ tons of coals last a month, how many tons will be required for a year?

$$\begin{array}{lrcl} & \text{Months.} & & \text{Tons.} \\ \text{Quantity of coals for} & 1 & = & 2\frac{1}{2} \\ \quad " & 12 & = & 2\frac{1}{2} \times 12 \end{array}$$

$2\frac{1}{2} \times 12 = 30$ tons. *Ans.*

(Ex. 160.)—If the price of a yard of velvet be 4/6, how much must I give for 28 inches?

1 yard = 36 inches.

$$\begin{array}{lrcl} & \text{Inches.} & & \\ \text{Price for} & 36 & = & 4/6 \\ \quad " & 1 & = & \dfrac{4/6}{36} \\ \quad " & 28 & = & \dfrac{4/6 \times 28}{36} \end{array}$$

$$\frac{4/6 \times \overset{7}{\cancel{28}}}{\underset{9}{\cancel{36}}} = \frac{31/6}{9} = 3/6. \quad \textit{Ans.}$$

(Ex. 161.)—If 2yds. 2qrs. 2nls. of cloth cost £1 10s. 10d. how much must be given for 30yds. 1qr. 1nl.?

2yds. 2qrs. 2nls. = 42nls.

30yds. 1qr. 1nl. = 485nls.

$$\begin{array}{lrcl} & \text{Nls.} & & \\ \text{Price for} & 42 & = & \text{£1 10s. 10d.} \\ \quad " & 1 & = & \dfrac{\text{£1 10s. 10d.}}{42} \\ \quad " & 485 & = & \dfrac{\text{£1 10s. 10d.} \times 485}{42} \end{array}$$

$$\frac{\text{£}\cancel{1}\ \overset{15}{\cancel{10}}\text{s.}\ \overset{5}{\cancel{10}}\text{d.} \times 485}{\underset{21}{\cancel{42}}} = \frac{\text{£373 17s. 1d.}}{21} = \text{£17 16s. } 0\tfrac{1}{2}\text{d.} + \tfrac{10}{21}\text{f.} \quad \textit{Ans.}$$

(Ex. 162.)—If one pound weight of gold be worth £46 14s. 6d., how much is that for 5 ounces?

1lb. Troy = 12oz.

$$\text{Value of } \overset{\text{Oz.}}{12} = £46\ 14\text{s.}\ 6\text{d.}$$

$$\text{,,} \quad 1 = \frac{£46\ 14\text{s.}\ 6\text{d.}}{12}$$

$$\text{,,} \quad 5 = \frac{£46\ 14\text{s.}\ 6\text{d.} \times 5}{12}$$

$$\frac{£46\ 14\text{s.}\ 6\text{d.} \times 5}{12} = \frac{£233\ 12\text{s.}\ 6\text{d.}}{12} = £19\ 9\text{s.}\ 4\tfrac{1}{2}\text{d.} \quad \textit{Ans.}$$

(Ex. 163.)—A gentleman with an income of £210 paid £2 0s. 6d. for income tax; how much will a person with an income of £450 pay?

£2 0s. 6d = 486d.

$$\text{Tax paid on } £210 = 486\text{d.}$$

$$\text{,,} \quad 1 = \frac{486}{210}$$

$$\text{,,} \quad 450 = \frac{486 \times 450}{210}$$

$$\frac{486 \times \overset{15}{\cancel{450}}}{\underset{7}{\cancel{210}}}\text{d.} = \frac{7290}{7}\text{d.} = 1041\tfrac{3}{7}\text{d.} = £4\ 6\text{s.}\ 9\tfrac{3}{7}\text{d; or}$$

$$= £4\ 6\text{s.}\ 9\tfrac{1}{4}\text{d.} + \tfrac{5}{7}. \quad \textit{Ans.}$$

(Ex. 164.)—If it takes 5,000 bricks 9 inches long to build a wall, how many will be required if the bricks are two inches longer?

Two inches longer = 11 inches.

$$\text{Number 9 in. long} = 5000$$

$$\text{,,} \quad 1 \text{ in. long} = 5000 \times 9$$

$$\text{,,} \quad 11 \text{ in. long} = \frac{5000 \times 9}{11}$$

$$\frac{5000 \times 9}{11} = \frac{45000}{11} = 4090\tfrac{10}{11} \text{ bricks.} \quad \textit{Ans.}$$

Note the unity line. If the bricks are only 1in. long it will take nine times as many as when they are 9in. long.

(Ex. 165.)—If 1 cwt. of Cheshire cheese costs £4 18s., how much must I give for $3\frac{1}{2}$ lbs?

1 cwt. = 112. £4 18s. = 98s.

$$\begin{array}{lrcl} & \text{lb.} & & \text{shil.} \\ \text{Cost of} & 112 & = & 98 \\ \quad ,, & 1 & = & \frac{98}{112} \\ \quad ,, & 3\frac{1}{2} & = & \frac{98 \times 3\frac{1}{2}}{112} \end{array}$$

$$\frac{98 \times 3\frac{1}{2}}{112}\text{s.} = \frac{\overset{49}{\cancel{98}} \times 7}{112 \times \cancel{2}}\text{s.} = \frac{343}{112}\text{s.} = 3\text{s. } 0\tfrac{3}{4}\text{d. } \textit{Ans.}$$

(Ex. 166.)—How many yards of cloth three-quarters wide are equal in measure to 30 yards five-quarters wide?

The Unity line requires careful thought, for there must be a greater length of one-quarter wide than of five-quarters wide.

$$\begin{array}{lrcl} & \text{qrs.} & & \text{yds.} \\ \text{Length at} & 5 & = & 30 \\ \quad ,, & 1 & = & 30 \times 5 \\ \quad ,, & 3 & = & \frac{30 \times 5}{3} \end{array}$$

$$\frac{\overset{10}{\cancel{30}} \times 5}{\cancel{3}} = 50 \text{ yards. } \textit{Ans.}$$

(Ex. 167.)—A man walks 60 yards in 30 seconds, how far can he go in half-an-hour?

Half-an-hour = 30 min. × 60 = 1800 seconds.

$$\begin{array}{lrcl} & \text{secs.} & & \text{yds.} \\ \text{Distance in} & 30 & = & 60 \\ \quad ,, & 1 & = & \frac{60}{30} \\ \quad ,, & 1800 & = & \frac{60 \times 1800}{30} \end{array}$$

$$\frac{\overset{2}{\cancel{60}} \times 1800}{\cancel{30}} = 3600\text{yds} = 2\text{mls. } 10\text{yds. } \textit{Ans.}$$

(Ex. 168.)—If $3\frac{1}{2}$ yards of merino cost 6/9, how much must be given for $10\frac{1}{2}$ yards?

$3\frac{1}{2}$yds = 7 half-yards. $10\frac{1}{2}$yds = 21 half-yards.

$$\begin{array}{rl} & \text{Hf.-yds.} \\ \text{Cost of} & 7 = 6/9 \\ \text{,,} & 1 = \dfrac{6/9}{7} \\ \text{,,} & 21 = \dfrac{6/9 \times 21}{7} \end{array}$$

$$\frac{6/9 \times \overset{3}{\cancel{21}}}{\cancel{7}} = 20/3 = £1\ 0s.\ 3d.\ \textit{Ans.}$$

(Ex. 169.)—If I give £4 18s. for 3 cwt. of sugar, at what rate do I buy it per dozen pounds?

3 cwt. = 112 × 3 = 336lb. £4 18s. = 98/-

$$\begin{array}{rl} & \text{lbs. Shill.} \\ \text{Price for} & 336 = 98 \\ \text{,,} & 1 = \dfrac{98}{336} \\ \text{,,} & 12 = \dfrac{98 \times 12}{336} \end{array}$$

$$\frac{98/- \times \cancel{12}}{\underset{28}{\cancel{336}}} = \frac{98/-}{28} = 3/6.\ \textit{Ans.}$$

(Ex. 170.)—If I bought 20 pieces of cloth, each 20 yards, at 12/- per ell (English), what is the value of 14 yards?

This sum is simply a "catch" question, and resolves itself into finding the price of 14 yards (or 56 quarter-yards), if 5 quarter-yards (*i.e.*, one E. ell) cost 12/-.

$$\begin{array}{rl} \text{Price for} & 5 \text{ qrs.} = 12/- \\ \text{,,} & 1 \text{ ,,} = \dfrac{12/-}{5} \\ \text{,,} & 56 \text{ ,,} = \dfrac{12/- \times 56}{5} \end{array}$$

$$\frac{12/- \times 56}{5} = \frac{672/-}{5} = 134\tfrac{2}{5}\text{s.} = £6\ 14\text{s.}\ 4\tfrac{3}{4}\text{d.} + \tfrac{1}{5}.\ (\text{or } 4\tfrac{4}{5}\text{d}).\ \textit{Ans.}$$

(Ex. 171.)—If 136 masons build a fort in 28 days, how many men would be able to build it in 8 days less?

8 days less = 28 − 8 = 20 days. The unity line requires attention.

$$\begin{array}{lcl} & \text{Days.} & \text{Men.} \\ \text{No. of men for} & 28 = & 136 \\ \quad ,, & 1 = & 136 \times 28 \\ \quad ,, & 20 = & \dfrac{136 \times 28}{20} \end{array}$$

$$\frac{136 \times \overset{7}{\cancel{28}}}{\underset{5}{\cancel{20}}} = \frac{952}{5} = 190\tfrac{2}{5} \text{ men. } \textit{Ans.}$$

(Ex. 172.)—If for 24s. I can have 1200lbs. carried 36 miles, how many pounds can I have carried 24 miles for the same money?

The 24s. being the same for either case does not enter into the question. The unity line will probably catch the children.

$$\begin{array}{lcl} & \text{Miles.} & \text{lbs.} \\ \text{Pounds for} & 36 = & 1200 \\ \quad ,, & 1 = & 1200 \times 36 \\ \quad ,, & 24 = & \dfrac{1200 \times 36}{24} \end{array}$$

$$\frac{\overset{600}{\cancel{1200}} \times \overset{3}{\cancel{36}}}{\underset{2}{\cancel{24}}} = 1800\text{lbs. } \textit{Ans.}$$

(Ex. 173.)—Find the cost of five cheeses, each weighing two stone, at 2s. 10d. for 4lb.

2 stone = 14lb. × 2 = 28lb.

$$\begin{array}{lcl} & \text{Lbs.} & \text{Pence.} \\ \text{Price of} & 4 = & 34 \\ \quad ,, & 1 = & \dfrac{34}{4} \\ \quad ,, & 28 = & \dfrac{34 \times 28}{4} \end{array}$$

$$\frac{34 \times \overset{7}{\cancel{28}}}{\cancel{4}}\text{d.} = 238\text{d.} = 19\text{s. } 10\text{d. price of one cheese.}$$

Then 19s. 10d. × 5 = £4 19s. 2d. ***Ans.***

(Ex. 174.)—If 3050 soldiers have provisions for 9 months, how long ought the same provisions to last 2000 men?

$$\begin{array}{lrcl} & \text{Soldiers.} & & \text{Months.} \\ \text{Time for} & 3050 & = & 9 \\ \quad ,, & 1 & = & 9 \times 3050 \\ \quad ,, & 2000 & = & \dfrac{9 \times 3050}{2000} \end{array}$$

$$\frac{9 \times \overset{61}{\cancel{3050}}}{\underset{40}{\cancel{2000}}} = \frac{549}{40} = 13\tfrac{29}{40} \text{ months. } \textit{Ans.}$$

(Ex. 175.)—What is the length of a pole throwing a shadow of 48 feet 3 inches, if the shadow of one 10 feet high be 9 feet 2 inches?

48ft. 3in. = 579in. 9ft. 2in. = 110 in.

$$\begin{array}{lrcl} & \text{In.} & & \text{Ft.} \\ \text{Height for a shadow of} & 110 & = & 10 \\ \quad ,, \qquad ,, & 1 & = & \dfrac{10}{110} \\ \quad ,, \qquad ,, & 579 & = & \dfrac{10 \times 579}{110} \end{array}$$

$$\frac{\cancel{10} \times 579}{\underset{11}{\cancel{110}}} = \frac{579}{11} = 52\tfrac{7}{11} \text{ feet. } \quad \textit{Ans.}$$

$$= 52 \text{ feet } 7\tfrac{7}{11}\text{in. } ,,$$

(Ex. 176.)—A bankrupt pays 12/8 in the pound, and his assets amount to £500. Find the amount of his debts?

The only difficulty is in the statement, which will be puzzling to children, chiefly because the question is out of the range of their ordinary experience.

12s. 8d. $= 12\frac{2}{3}$s. £500 $=$ 10000s.

$$\begin{array}{lrl} & \text{Shil.} & \text{£} \\ \text{Debt paid for by} & 12\frac{2}{3} = & 1 \\ \quad ,, & 1 = & \dfrac{1}{12\frac{2}{3}} \\ \quad ,, & 10000 = & \dfrac{1 \times 10000}{12\frac{2}{3}} \end{array}$$

$$£\frac{1 \times 10000}{12\frac{2}{3}} = £\frac{\frac{1}{1} \times \frac{10000}{1}}{\frac{38}{3}} = £\frac{10000 \times 3}{38} = £\frac{30000}{38}$$

$= £789$ 9s. $5\frac{1}{2}$d $+ \frac{14}{19}$ (or $5\frac{13}{19}$d.). *Ans.*

(Ex. 177.)—A man laid out £50 in eggs, buying them at 10d. per score. What did he gain by selling them at 2d. each?

First find number of eggs bought?

£50 $= 50 \times 20 \times 12 = 12000$d.

$$\begin{array}{lrl} & \text{Pence.} & \text{Eggs.} \\ \text{No of eggs for} & 10 = & 20 \\ \quad ,, & 1 = & \dfrac{20}{10} \\ \quad ,, & 12000 = & \dfrac{20 \times 12000}{10} \end{array}$$

$$\frac{\overset{2}{\cancel{20}} \times 12000}{\cancel{10}} = 24000 \text{ eggs.}$$

Selling price at 2d. each $= 24000 \times 2 = 48000$d. $=$ £200.

$\therefore$ gain $=$ £200 $-$ £50 $=$ £150. *Ans.*

(Ex. 178.)—A bankrupt's debts amount to £3548 6s. 8d., what will his creditors lose if he pays 12s. $10\frac{1}{2}$d. in the pound.

Loss on £1 $=$ £1 $-$ 12s. $10\frac{1}{2} =$ 7s. $1\frac{1}{2}$d.

£3548 6s. 8d. $=$ £$3548\frac{1}{3}$

Loss on £1 $=$ 7s. $1\frac{1}{2}$d.

,, £$3548\frac{1}{3} =$ 7s. $1\frac{1}{2}$d. $\times 3548\frac{1}{3}$

7s. $1\frac{1}{2} \times 3548\frac{1}{3} =$ £1264 1s. $10\frac{1}{2}$d. *Ans.*

The best way to work this out is by practice, thus—

$$\begin{array}{l|l|rr}
1\frac{1}{2}\text{d.} & \frac{1}{8} & 3548 & \\
 & & 7 & \\
\hline
 & & 24836 & \\
 & & 443 & 6 \\
 & & 2 & 4\frac{1}{2} \\
\hline
 & 2,0 & 2528,1 & 10\frac{1}{2} \\
\hline
 & & \pounds 1264 \quad 1 & 10\frac{1}{2}
\end{array} \quad \textit{Ans.}$$

$$\begin{array}{r|rr}
 & \text{s.} & \text{d.} \\
3 & 7 & 1\frac{1}{2} \\
\hline
 & 2 & 4\frac{1}{2}
\end{array} = \frac{1}{3} \text{ of 7s. } 1\frac{1}{2}\text{d.}$$

(Ex. 179.)—I buy 8lbs. of butter for 13s. 4d. How much shall I get for two pounds?

$$\pounds 2 = 2 \times 20 \times 12 = 480\text{d.} \quad 13\text{s. } 4\text{d.} = 160\text{d.}$$

$$\begin{array}{lrl}
 & \text{Pence.} & \text{lbs.} \\
\text{Pounds for} & 160 = & 8 \\
\quad ,, & 1 = & \frac{8}{160} \\
\quad ,, & 480 = & \frac{8 \times 480}{160}
\end{array}$$

$$\frac{8 \times \overset{3}{\cancel{480}}}{\cancel{160}} = 24\text{lbs.} \quad \textit{Ans.}$$

(Ex. 180.)—If 6 men build a wall 60 yards long in 10 days of 8 hours each, how long will they be in building it if they work twice as hard for 12 hours a day?

As the men and the yards are the same in both cases, this reduces the sum to one in simple proportion.

$$\begin{array}{lrl}
 & \text{Hrs.} & \text{Days.} \\
\text{Time at} & 8 \text{ per day} = & 10 \\
\quad ,, & 1 \quad ,, \quad = & 10 \times 8 \\
\quad ,, & 12 \quad ,, \quad = & \frac{10 \times 8}{12}
\end{array}$$

But if they work twice as hard, they will be only half as long.

$$\therefore \frac{10 \times \overset{2}{\cancel{8}}}{\underset{3}{\cancel{12}} \times \cancel{2}} = \frac{10}{3} = 3\frac{1}{3} \text{ days.} \quad \textit{Ans.}$$

(Ex. 181.)—If 10 men or 20 boys can do a piece of work in 6 days, how long will it take 5 men and 30 boys to do it?

This resolves itself into two sums in Simple Rule of Three. First bring the boys to men, and see how many men the boys and men are equal to, then find the time.

$$\begin{aligned} &\quad\text{Boys. Men.}\\ &\text{If } 20=10\\ &\text{then } 1=\frac{10}{20}\\ &\text{and } 30=\frac{10\times 30}{20}=15 \text{ men.}\\ &\therefore 5 \text{ men}+15 \text{ men}=20 \text{ men.}\end{aligned}$$

$$\begin{aligned} &\quad\text{Men. Days.}\\ &\text{Time for } 10=6\\ &\quad ,, \quad 1=6\times 10\\ &\quad ,, \quad 20=\frac{6\times 10}{20}\end{aligned}$$

$$\frac{6\times \cancel{10}}{\underset{2}{\cancel{20}}}=3 \text{ days. } \textit{Ans.}$$

(Ex. 182.)—If 300lb. of tea cost £15, how much is $15\frac{1}{2}$lb. worth? How many lbs. of butter at 2s. per lb. could be bought for the worth of this tea?

$$\begin{aligned} &\qquad\quad\text{lbs.} \quad £\\ &\text{Value of } 300=15\\ &\quad ,, \qquad 1=\frac{15}{300}\\ &\quad ,, \quad 15\tfrac{1}{2}=\frac{15\times 15\frac{1}{2}}{300}\end{aligned}$$

$$\text{Then } £\frac{15\times 15\frac{1}{2}}{300}=£\frac{\cancel{15}\times 31}{\underset{20}{\cancel{300}}\times 2}=£\frac{31}{40}=15/6. \quad \textit{Ans.}$$

$$\text{Then } 15/6\div 2=\frac{15\frac{1}{2}}{2}=\frac{31}{2\times 2}=\frac{31}{4}=7\tfrac{3}{4}\text{lbs.} \quad \textit{Ans.}$$

(Ex. 183.)—If 9 men mow a field of 6ac. 2rds. in 20 days, how many men will be required to mow 3 fields of 9ac. 2rds. each, working at the same rate?

6ac. 2rds. = 26rds. 9ac. 2rds. × 3 = 38rds. × 3 = 114rds.

$$\begin{array}{lrl} & \text{Rds.} & \text{Men.} \\ \text{Men to mow} & 26 = & 9 \\ \quad " & 1 = & \dfrac{9}{26} \\ \quad " & 114 = & \dfrac{9 \times 114}{26} \end{array}$$

$$\frac{9 \times 114}{26} = \frac{1026}{26} = 39\tfrac{6}{13} \text{ men.} \qquad \textit{Ans.}$$

(Ex. 184.)—If $5\frac{3}{5}$ yards cost £$4\frac{1}{10}$, find the cost of $3\frac{3}{7}$ yards.

$$\begin{array}{lrl} & \text{Yds.} & £ \\ \text{Cost of} & 5\frac{3}{5} = & 4\frac{1}{10} \\ \quad " & 1 = & \dfrac{4\frac{1}{10}}{5\frac{3}{5}} \\ \quad " & 3\frac{3}{7} = & \dfrac{4\frac{1}{10} \times 3\frac{3}{7}}{5\frac{3}{5}} \end{array}$$

$$\frac{£4\frac{1}{10} \times 3\frac{3}{7}}{5\frac{3}{5}} = \frac{£\frac{41}{10} \times \frac{24}{7}}{\frac{28}{5}} = £\frac{41 \times \overset{6}{\cancel{24}} \times \cancel{5}}{\underset{2}{\cancel{10}} \times 7 \times \underset{7}{\cancel{28}}} = £\frac{246}{98} = £2\ 10s.\ 2\tfrac{1}{4}d. + \tfrac{39}{49}. \quad \textit{Ans.}$$

Or, £2 10s. $2\frac{22}{49}$d. *Ans.*

(Ex. 185.)—If a servant's wages for 20 weeks be £3 10s., how long ought he to serve for 20 guineas?

20gs. = 420/-. £3 10s. = 70/-.

$$\begin{array}{lrl} & \text{Shil.} & \text{Wks.} \\ \text{Time for} & 70 = & 20 \\ \quad " & 1 = & \dfrac{20}{70} \\ \quad " & 420 = & \dfrac{20 \times 420}{70} \end{array}$$

$$\frac{20 \times \overset{6}{\cancel{420}}}{\cancel{70}} = 20 \times 6 = 120 \text{ weeks} = 2 \text{ years } 16 \text{ weeks.} \quad \textit{Ans.}$$

(Ex. 186.)—If a man travels $13\frac{3}{4}$ hours a day and performs a journey in $27\frac{5}{8}$ days, in how many days will he do the same journey if he travels $10\frac{2}{7}$ hours per day?

$$\begin{array}{llll} & \text{Hours.} & & \text{Days.} \\ \text{Time to travel at} & 13\frac{3}{4} & \text{per day} & =27\frac{5}{8} \\ \quad " & 1 & \quad " & =27\frac{5}{8}\times 13\frac{3}{4} \\ \quad " & 10\frac{2}{7} & \quad " & =\dfrac{27\frac{5}{8}\times 13\frac{3}{4}}{10\frac{2}{7}} \end{array}$$

$$\frac{27\frac{5}{8}\times 13\frac{3}{4}}{10\frac{2}{7}}=\frac{\frac{221}{8}\times\frac{55}{4}}{\frac{72}{7}}=\frac{221\times 55\times 7}{8\times 4\times 72}=\frac{85085}{2304}=36\tfrac{2141}{2304}\text{ days. } \textit{Ans.}$$

(Ex. 187.)—If 12 men earn £68·5 in 4·6 weeks, how many men will earn £85·625 in 6·9 weeks?

$$\begin{array}{llll} & \text{£} & \text{Wks.} & \text{Men.} \\ \text{Men to earn} & 68{\cdot}5 & \text{in } 4{\cdot}6 & =12 \\ \quad " & 1 & \;" \; 4.6 & \\ \quad " & 1 & \;" \; 1 & \\ \quad " & 85.625 & \;" \; 1 & \\ \quad " & 85.525 & \;" \; 6.9 & \end{array}\left.\vphantom{\begin{array}{l}1\\1\\1\\1\end{array}}\right\}=\frac{12\times 4.6\times 85.625}{68.5\times 6.9}$$

$$\frac{\overset{4}{\cancel{12}}\times\overset{2}{\cancel{4.6}}\times\overset{1.25}{\cancel{85.625}}}{\cancel{68.5}\times\underset{2.3}{\cancel{6.9}}}=8\times 1.25=10\text{ men. } \textit{Ans.}$$

(Ex. 188.)—A man's income is reduced from £698 to £677 12s. 10d. by payment of Income Tax, how much does he pay in the pound?

$$£698-£677\ 12\text{s. }10\text{d.}=£20\ 7\text{s. }2\text{d.}$$

$$\begin{array}{lrl} & \text{£} & \text{£ s. d.} \\ \text{Tax on} & 698 & =20\ \ 7\ \ 2 \\ \quad " & 1 & =\dfrac{20\ \ 7\ \ 2}{698}=7\text{d. } \textit{Ans.} \end{array}$$

(Ex. 189.)—Find the value of 17 articles at £19 16s. for every 13.

$$\begin{array}{lrl} & \text{Articles.} & \text{£ s. d.} \\ \text{Value of} & 13 & =19\ 16\ 0 \\ \quad " & 1 & =\dfrac{19\ 16\ 0}{13} \\ \quad " & 17 & =\dfrac{19\ 16\ 0\times 17}{13} \end{array}$$

$$\frac{£19\ 16\text{s. }0\text{d.}\times 17}{13}=\frac{£336\ 12\text{s. }0\text{d.}}{13}=£25\ 17\text{s. }10\tfrac{2}{13}\text{d. } \textit{Ans.}$$

(Ex. 190.)—If the 4lb. loaf costs $8\frac{1}{2}$d. when wheat is 56s. a quarter, what should be its weight when wheat is 52s. a quarter?

As $8\frac{1}{2}$d. is the price in both cases, it does not enter into the question, which is, therefore, a Simple Rule of Three.

$$\begin{array}{lllll} & & \text{Shil.} & & \text{lb.} \\ \text{Weight of loaf with wheat at} & & 56 & \text{per qr.} & =4 \\ \quad ,, & ,, & 1 & ,, & =4\times 56 \\ \quad ,, & ,, & 52 & ,, & =\dfrac{4\times 56}{52} \end{array}$$

$$\frac{\cancel{4}\times 56}{\underset{13}{\cancel{52}}}=4\tfrac{4}{13}\text{lb., or 4lb. 4oz. }14\tfrac{10}{13}\text{drs.}\quad \textit{Ans.}$$

(Ex. 191.)—If 5 horses are worth 7 cows, and 4 cows cost £50, what is the value of 6 horses?

$$\begin{array}{lrl} & \text{Horses.} & \text{Cows.} \\ \text{Value of} & 5 & =7 \\ \quad ,, & 1 & =\dfrac{7}{5} \\ \quad ,, & 6 & =\dfrac{7\times 6}{5}=\dfrac{42}{5}\text{ cows.} \end{array}$$

$$\begin{array}{lrl} & \text{Cows.} & £ \\ \text{Value of} & 4 & =50 \\ \quad ,, & 1 & =\dfrac{50}{4} \\ \quad ,, & \dfrac{42}{5} & =\dfrac{50}{4}\times\dfrac{42}{5} \end{array}$$

$$\frac{£\overset{10}{\cancel{50}}\times\overset{21}{\cancel{42}}}{\underset{2}{\cancel{4}}\times\cancel{5}}=\frac{£210}{2}=£105.\quad \textit{Ans.}$$

(Ex. 192.)—If 7 bags of rice, weighing 516lb., are conveyed 200 miles for £13 14s., what will be the cost of conveying 430lb. the same distance?

As the distance is the same in both cases, it does not enter into the question.

$$\begin{array}{llrl} & & \text{lb.} & \quad\text{£}\quad\text{s.} \\ \text{Cost of carrying} & & 516 & = 13\ \ 14 \\ \quad ,, & ,, & 1 & = \dfrac{\text{£}13\ 14\text{s.}}{516} \\ \quad ,, & ,, & 430 & = \dfrac{\text{£}13\ 14\text{s.} \times 430}{516} \end{array}$$

$$\frac{\text{£}13\ 14\text{s.} \times \overset{215}{\cancel{430}}}{\underset{258}{\cancel{516}}} = \frac{\text{£}2945\ 10\text{s.}}{258} = \text{£}11\ 8\text{s.}\ 4\text{d.}\quad \textit{Ans.}$$

(Ex. 193.)—What would be the railway fare for a distance of 26 miles, if I can go 65 miles for 10 shillings?

$$\begin{array}{lrl} & \text{Miles.} & \text{Shil.} \\ \text{Fare for} & 65 & = 10 \\ \quad ,, & 1 & = \dfrac{10}{65} \\ \quad ,, & 26 & = \dfrac{10 \times 26}{65} \end{array}$$

$$\frac{\overset{2}{\cancel{10}} \times \overset{2}{\cancel{26}}}{\underset{\cancel{13}}{\cancel{65}}} = 2 \times 2 = 4\text{s.}\quad \textit{Ans.}$$

COMPOUND PROPORTION.

(Ex. 194.)—If the wages of 10 men for half a year be £30, what will be the wages of 14 men for 14 weeks?

$$\begin{array}{lrcrl} & \text{Men.} & & \text{Weeks.} & \\ \text{Wages for} & 10 & \text{for} & 26 & = \text{£}30 \\ \quad ,, & 1 & ,, & 26 & \\ \quad ,, & 1 & ,, & 1 & \\ \quad ,, & 14 & ,, & 1 & \\ \quad ,, & 14 & ,, & 14 & \end{array}\left.\vphantom{\begin{array}{l}1\\1\\1\\1\end{array}}\right\} = \frac{\text{£}30 \times 14 \times 14}{10 \times 26}$$

$$\frac{\text{£}\overset{3}{\cancel{30}} \times \overset{7}{\cancel{14}} \times 14}{\cancel{10} \times \underset{13}{\cancel{26}}} = \frac{\text{£}3 \times 98}{13} = \frac{\text{£}294}{13} = \text{£}22\ 12\text{s.}\ 3\tfrac{1}{2}\text{d.} + \tfrac{10}{13}\ (\text{or } 3\tfrac{9}{13}\text{d.})\quad \textit{Ans.}$$

(Ex. 195.)—If £150 gains £2 5s. in 4 months, what sum will gain £2 1s. in 11 months?

£2 5s. = 45s. £2 1s. = 41s.

	Shil.		Months.		£
Sum to gain	45	in	4	=	150
"	1	"	4	=	$\frac{150}{45}$
"	1	"	1	=	$\frac{150 \times 4}{45}$
"	41	"	1	=	$\frac{150 \times 4 \times 41}{45}$
"	41	"	11	=	$\frac{150 \times 4 \times 41}{45 \times 11}$

$$\frac{\pounds\overset{\overset{10}{\cancel{30}}}{\cancel{150}} \times 4 \times 41}{\underset{\underset{3}{\cancel{9}}}{\cancel{45}} \times 11} = \frac{\pounds 10 \times 164}{33} = \frac{\pounds 1640}{33} = \pounds 49\ 13s.\ 11\tfrac{1}{4}d. + \tfrac{1}{44}\ (\text{or } 11\tfrac{3}{11}d.)$$

Ans.

(Ex. 196.)—The penny roll weighs 8oz. when wheat is 14s. a bushel. How much should the 2d. loaf weigh when wheat is 19s. a bushel?

	Loaf.		Shil		Oz.
Weight of	1d.	with wheat at	14	=	8
"	1d.	"	1	=	8×14
"	2d.	"	1	=	$8 \times 14 \times 2$
"	2d.	"	19	=	$\frac{8 \times 14 \times 2}{19}$

$$\frac{8 \times 14 \times 2}{19} = \frac{224}{19} = 11\tfrac{15}{19}\text{oz.}$$ *Ans.*

(Ex. 197.)—If 4 men in 3 days earn 33s., how many men will earn 3 times as much in 5 days?

Three times as much = 33s. × 3

	Shil.		Days.		Men.
Men to earn	33	in	3	=	4
"	1	"	3		
"	1	"	1	=	$\frac{4 \times 3 \times (33 \times 3)}{33 \times 5}$
"	(33 × 3)	"	1		
"	(33 × 3)	"	5		

$$\frac{4 \times 3 \times \cancel{33} \times 3}{\cancel{33} \times 5} = \frac{36}{5} = 7\tfrac{1}{5}\text{ men.}$$ *Ans.*

(Ex. 198.)—Three men in $10\frac{1}{2}$ days earn £9 14s. 3d. How many men will earn £45 15s. 9d. in $5\frac{1}{2}$ days?

£9 14s. 3d. = 2331d. £45 15s. 9d. = 10989d.

	Pence.		Days.		Men.
Men to earn	2331	in	$10\frac{1}{2}$	=	3
„	1	„	$10\frac{1}{2}$	=	$\frac{3}{2331}$
„	1		1	=	$\frac{3\times 10\frac{1}{2}}{2331}$
„	10989	„	1	=	$\frac{3\times 10\frac{1}{2}\times 10989}{2331}$
„	10989	„	$5\frac{1}{2}$	=	$\frac{3\times 10\frac{1}{2}\times 10989}{2331\times 5\frac{1}{2}}$

$$\frac{\cancel{3}\times\cancel{10\frac{1}{2}}\times 10989}{\underset{\substack{\cancel{777}\\74}}{\cancel{2331}}\times 5\frac{1}{2}}=\frac{10989}{407}=27 \text{ men. } \textit{Ans.}$$

(Ex. 199.)—If it takes 21 men 5 days to mow 72 acres of grass, how many men must be employed to mow 320a. 2r. 4pol. in 3 days?

72a. = 11520 pol. 320a. 2r. 4pol. = 51284 pol.

	Pol.		Days.		Men.
Men to mow	11520	in	5		= 21
„	1	„	5		
„	1	„	1		$=\frac{21\times 5\times 51284}{11520\times 3}$
„	51284	„	1		
„	51284	„	3		

$$\frac{\overset{7}{\cancel{21}}\times\cancel{5}\times\overset{12821}{\cancel{51284}}}{\underset{\substack{\cancel{2304}\\576}}{\cancel{11520}}\times\cancel{3}}=\frac{89747}{576}=155\tfrac{467}{576}\text{ men. } \textit{Ans.}$$

(Ex. 200.)—Nine persons spend £60 in 4 months: how much will be required by 13 people for 7 months?

	Persons.		Mths.		£
Amount spent by	9	in	4	=	60
"	1	"	4	=	$\frac{60}{9}$
"	1	"	1	=	$\frac{60}{9\times 4}$
"	13	"	1	=	$\frac{60\times 13}{9\times 4}$
"	13	"	7	=	$\frac{60\times 13\times 7}{9\times 4}$

$$\frac{\pounds\cancel{60}^{\cancel{15}\,5}\times 13\times 7}{\cancel{9}_{3}\times\cancel{4}}=\frac{\pounds 65\times 7}{3}=\frac{\pounds 455}{3}=\pounds 151\tfrac{2}{3}=\pounds 151\ 13s.\ 4d.\quad \textit{Ans.}$$

(Ex. 201.)—How long will it take 17 men to earn £50 if 20 men earn 13 guineas in $6\frac{1}{2}$ days?

£50 = 1000s. 13 guineas = 273s.

	Men.		Shil.		Days.
Time for	20	to earn	273	=	$6\frac{1}{2}$
"	1	"	273		
"	1	"	1	=	$\frac{6\frac{1}{2}\times 20\times 1000}{273\times 17}$
"	17	"	1		
"	17	"	1000		

$$\frac{\cancel{6\tfrac{1}{2}}\times\cancel{20}^{10}\times 1000}{\cancel{273}_{\cancel{42}\,21}\times 17}=\frac{10000}{357}=28\tfrac{4}{357}\text{ days.}\quad \textit{Ans.}$$

(Ex. 202.)—If 300 men can do a piece of work in 24 days, how many men can do one-third as much in 12 days?

This may be worked as a simple Rule of Three.

	Days.	Men.
No. of men to do the work in	24	= 300
"	1	= 300 × 24
"	12	= $\frac{300\times 24}{12}$

But the work is only $\frac{1}{3}$ as great.

$$\therefore \text{ men to do } \tfrac{1}{3} \text{ the work} = \frac{300 \times 24}{12 \times 3}$$

$$\frac{\overset{100}{\cancel{300}} \times \overset{2}{\cancel{24}}}{\cancel{12} \times \cancel{3}} = 200 \text{ men. } \textit{Ans.}$$

(Ex. 203.)—A party of 7 gentlemen on a journey together spend £150 in 3 weeks 4 days, what would be the expenses of 11 persons for 14 days at the same rate?

3 weeks 4 days = 25 days.

$$\begin{array}{lccl} & \text{Men.} & \text{Days.} & \text{£} \\ \text{Expenses for} & 7 & \text{for } 25 & = 150 \\ \quad ,, & 1 & ,, \ 25 & \\ \quad ,, & 1 & ,, \ 1 & \\ \quad ,, & 11 & ,, \ 1 & \\ \quad ,, & 11 & ,, \ 14 & \end{array} \Bigg\} = \frac{150 \times 11 \times 14}{7 \times 25}$$

$$\frac{\overset{6}{\cancel{150}} \times 11 \times \overset{2}{\cancel{14}}}{\cancel{7} \times \cancel{25}} = \text{£}66 \times 2 = \text{£}132. \quad \textit{Ans.}$$

(Ex. 204.)—If 125 yards of flannel, 6 quarters wide, cost £28 2s. 6d., what should be paid for 350 yards $2\frac{1}{2}$ yards wide?

$2\frac{1}{2}$yds. = 10qrs. £28 2s. 6d. = £$28\frac{1}{8}$.

$$\begin{array}{llll} \text{Cost of} & 125\text{yds} & 6\text{qrs wide} & = \text{£}28\tfrac{1}{8} \\ \quad ,, & 1 \ ,, & 6 \quad ,, & = \dfrac{28\frac{1}{8}}{125} \\ \quad ,, & 1 \ ,, & 1 \quad ,, & = \dfrac{28\frac{1}{8}}{125 \times 6} \\ \quad ,, & 350 \ ,, & 1 \quad ,, & = \dfrac{28\frac{1}{8} \times 350}{125 \times 6} \\ \quad ,, & 350 \ ,, & 10 \quad ,, & = \dfrac{28\frac{1}{8} \times 350 \times 10}{125 \times 6} \end{array}$$

$$\frac{28\frac{1}{8} \times \overset{\overset{7}{\cancel{14}}}{\cancel{350}} \times \overset{2}{\cancel{10}}}{\underset{\cancel{5}}{\cancel{125}} \times \underset{3}{\cancel{6}}} = \text{£}\frac{196\frac{7}{8} \times 2}{3} = \text{£}\frac{393\frac{3}{4}}{3} = \text{£}131\tfrac{1}{4} = \text{£}131 \ 5\text{s.} \quad \textit{Ans.}$$

(Ex. 205.)—If it costs £59 2s. 1½d. to keep 3 horses for seven months, what will it cost to keep 2 horses for 11 months?

$$\begin{array}{lcccl} & \text{Horses.} & & \text{Months.} & \\ \text{Cost of} & 3 & \text{for} & 7 = & £59\ 2\text{s}.\ 1\tfrac{1}{2}\text{d}. \\ \quad " & 1 & " & 7 & \\ \quad " & 1 & " & 1 & \\ \quad " & 2 & " & 1 & \\ \quad " & 2 & " & 11 & \end{array} \left.\begin{array}{c} \\ \\ \\ \\ \end{array}\right\} = \frac{£59\ 2\text{s}.\ 1\tfrac{1}{2}\text{d}. \times 2 \times 11}{3 \times 7}$$

$$\frac{£59\ 2\text{s}.\ 1\tfrac{1}{2}\text{d}. \times 22}{21} = \frac{£1300\ 6\text{s}.\ 9\text{d}.}{21} = £61\ 18\text{s}.\ 5\text{d}. \quad \textit{Ans.}$$

(Ex. 206.)—If the carriage of 8cwt. of goods for 124 miles be 6 guineas, how much ought to be carried 53 miles for half the money?

Half the money = 3 guineas.

$$\begin{array}{lccccl} & \text{Gs.} & & \text{Mls.} & & \text{Cwt.} \\ \text{Weight carried for} & 6 & \text{for} & 124 & = & 8 \\ \quad " \qquad " & 1 & " & 124 & = & \frac{8}{6} \\ \quad " \qquad " & 1 & " & 1 & = & \frac{8 \times 124}{6} \\ \quad " \qquad " & 3 & " & 1 & = & \frac{8 \times 124 \times 3}{6} \\ \quad " \qquad " & 3 & " & 53 & = & \frac{8 \times 124 \times 3}{6 \times 53} \end{array}$$

$$\frac{8 \times \overset{62}{\cancel{124}} \times \cancel{3}}{\underset{\cancel{2}}{\cancel{6}} \times 53} = \frac{496}{53} = 9\tfrac{19}{53}\text{cwt., or 9cwt. 1qr. } 12\tfrac{8}{53}\text{lb.} \quad \textit{Ans.}$$

(Ex. 207.)—Ten men can reap a field of 7½ acres in 3 days of 12 hours each; how long will it take 8 men to reap 9 acres, working 8 hours a day?

Number of days for—

$$\begin{array}{cccccl} \text{Men.} & & \text{Acres.} & & \text{Hrs.} & \qquad \text{Days.} \\ 10 & \text{to reap} & 7\tfrac{1}{2} & \text{at} & 12 & \text{per day} = 3 \\ 1 & " & 7\tfrac{1}{2} & " & 12 & \quad " \\ 1 & " & 1 & " & 12 & \quad " \\ 1 & " & 1 & " & 1 & \quad " \\ 8 & " & 1 & " & 1 & \quad " \\ 8 & " & 9 & " & 1 & \quad " \\ 8 & " & 9 & " & 8 & \quad " \end{array} \left.\begin{array}{c} \\ \\ \\ \\ \\ \\ \end{array}\right\} = \frac{3 \times 10 \times 12 \times 9}{7\tfrac{1}{2} \times 8 \times 8}$$

$$\frac{3 \times \overset{5}{\cancel{10}} \times \overset{3}{\cancel{12}} \times 9}{7\tfrac{1}{2} \times \underset{4}{\cancel{8}} \times 8} = \frac{45 \times 9}{60} = \frac{405}{60} = 6\tfrac{45}{60} = 6\tfrac{3}{4} \text{ days.} \quad \textit{Ans.}$$

But $\tfrac{3}{4}$ of a day of 8 hours = 6 hours. ∴ time = 6 days 6 hours. *Ans.*

(Ex. 208.)—If 11 men can do a piece of work in 25 days, how many men will it take to do 7 times as much in one-fifth of the time?

$\frac{1}{5}$ the time $=25 \div 5=5$ days.

	Days.		Men.
Men to do 1 piece of work in	25	=	11
" 1 "	1	=	11×25
" 7 "	1	=	$11 \times 25 \times 7$
" 7 "	5	=	$\frac{11 \times 25 \times 7}{5}$

$$\frac{11 \times \overset{5}{\cancel{25}} \times 7}{\cancel{5}} = 55 \times 7 = 385 \text{ men. } \textit{Ans.}$$

(Ex. 209.)—If 42 yards of cloth, 18in. wide, cost £48 14s., what will $118\frac{1}{2}$ yards of cloth one yard wide cost?

£48 14s. $=$ £$48\frac{7}{10}$. 1 yard $=36$ inches.

	Yards.		Inches.	
Cost of	42	of cloth	18	wide $=$ £$48\frac{7}{10}$
"	1	"	18	"
"	1	"	1	"
"	$118\frac{1}{2}$	"	1	"
"	$118\frac{1}{2}$	"	36	"

$$\left.\right\} = \frac{£48\frac{7}{10} \times 118\frac{1}{2} \times 36}{42 \times 18}$$

$$\frac{£48\frac{7}{10} \times 118\frac{1}{2} \times 36}{42 \times 18} = \frac{£487 \times 237 \times \overset{2}{\cancel{36}}}{10 \times \cancel{2} \times 42 \times \cancel{18}} = \frac{£115419}{420} =$$

£274 16s. $1\frac{1}{2}$d. $+\frac{6}{7}$ (or $1\frac{5}{7}$d.). *Ans.*

(Ex. 210.)—If 16 men eat 14s. worth of bread in 4 days, how many men can be kept on £4 5s. $3\frac{3}{4}$d. for 10 days?

£4 5s. $3\frac{3}{4}$d. $=85\frac{5}{16}$s.

	Shil.		Days.		Men.
Men kept on	14	for	4	=	16
"	1	"	4	=	$\frac{16}{14}$
"	1	"	1	=	$\frac{16 \times 4}{14}$
"	$85\frac{5}{16}$	"	1	=	$\frac{16 \times 4 \times 85\frac{5}{16}}{14}$
"	$85\frac{5}{16}$	"	10	=	$\frac{16 \times 4 \times 85\frac{5}{16}}{14 \times 10}$

$$\frac{16 \times 4 \times 85\frac{5}{16}}{14 \times 10} = \frac{\cancel{16} \times \overset{2}{\cancel{4}} \times \overset{\overset{39}{\cancel{195}}}{\cancel{1365}}}{\underset{7}{\cancel{14}} \times \underset{5}{\cancel{10}} \times \cancel{16}} = 39 \text{ men. } \textit{Ans.}$$

(Ex. 211.)—If the carriage of 5cwt. for 75 miles be £9 6s., what will it cost to carry 9cwt. 25 miles?

	Cwt.		Miles.	
Cost for	5	for	75	= £9 6s.
„	1	„	75	
„	1	„	1	$= \dfrac{£9\ 6s. \times 9 \times 25}{5 \times 75}$
„	9	„	1	
„	9	„	25	

$$\frac{£9\ 6s. \times \overset{3}{\cancel{9}} \times \cancel{25}}{5 \times \underset{3}{\cancel{75}}} = \frac{£9\ 6s. \times 3}{5} = \frac{£27\ 18s.}{5} = £5\ 11s.\ 7d. + \tfrac{1}{5}\ (\text{or } 7\tfrac{1}{5}d).\ \textit{Ans.}$$

(Ex. 212.)—Thirteen horses plough 17 acres of land in 7 days. How many horses will be able to plough 69 acres in 19 days?

	Acres.		Days.	Horses.
Horses to plough	17	in	7	= 13
„	1	„	7	$= \dfrac{13}{17}$
„	1	„	1	$= \dfrac{13 \times 7}{17}$
„	69	„	1	$= \dfrac{13 \times 7 \times 69}{17}$
„	69	„	19	$= \dfrac{13 \times 7 \times 69}{17 \times 19}$

$$\frac{13 \times 7 \times 69}{17 \times 19} = \frac{6279}{323} = 19\tfrac{142}{323}\ \text{horses.}\quad \textit{Ans.}$$

(Ex. 213.)—If the railway fare for 28 persons for 56 miles be £7 11s. 1d., how much will it be for 16 persons going a distance of 28 miles?

	Persons.		Miles.	
Fare for	28	for	56	= £7 11s. 1d.
„	1	„	56	
„	1	„	1	$= \dfrac{£7\ 11s.\ 1d. \times 16 \times 28}{28 \times 56}$
„	16	„	1	
„	16	„	28	

$$\frac{£7\ 11s.\ 1d. \times \overset{2}{\cancel{16}} \times \cancel{28}}{\cancel{28} \times \underset{7}{\cancel{56}}} = \frac{£15\ \ 2s.\ 2d.}{7} = £2\ 3s.\ 2d.\quad \textit{Ans.}$$

(Ex. 214.)—If 24 men build 4 houses in 32 days, how many men can build 12 houses, twice as large, in 16 days?

	Houses.		Days.	Men.
Men to build	4	in	32	$=24$
"	1	"	32	$=\frac{24}{4}$
"	1	"	1	$=\frac{24\times 32}{4}$
"	12	"	1	$=\frac{24\times 32\times 12}{4}$
"	12	"	16	$=\frac{24\times 32\times 12}{4\times 16}$

$$\frac{\overset{6}{\cancel{24}}\times\overset{2}{\cancel{32}}\times 12}{\cancel{4}\times\cancel{16}}=12\times 12=144 \text{ men.}$$

But the houses are twice as large, $\therefore$ $144\times 2=288$ men. *Ans.*

(Ex. 215.)—If 90 yards of carpet, three-quarters of a yard wide, costs £13, how much must be given for 35 yards, half a yard wide?

	Yds.		Qrs.		
Cost of	90	carpet	3	wide	$=$ £13
"	1	"	3	"	
"	1	"	1	"	$=\frac{£13\times 35\times 2}{90\times 3}$
"	35	"	1	"	
"	35	"	2	"	

$$\frac{£13\times\overset{7}{\cancel{35}}\times\cancel{2}}{\underset{\underset{9}{\cancel{45}}}{\cancel{90}}\times 3}=\frac{£91}{27}=£3\ 7s.\ 4\tfrac{4}{9}d.+\tfrac{5}{9}\ (\text{or } 4\tfrac{8}{9}d).\ \textit{Ans.}$$

(Ex. 216.)—If 13 men reap 16 acres in $12\frac{1}{2}$ hours, how long will it take 12 men to reap 19 acres?

$$\begin{array}{lccccl} & \text{Men.} & & \text{Acres.} & \text{Hrs.} & \\ \text{Time for} & 13 & \text{to reap} & 16 & = & 12\tfrac{1}{2} \\ \quad " & 1 & " & 16 & = & 12\tfrac{1}{2}\times 13 \\ \quad " & 1 & " & 1 & = & \dfrac{12\frac{1}{2}\times 13}{16} \\ \quad " & 12 & " & 1 & = & \dfrac{12\frac{1}{2}\times 13}{16\times 12} \\ \quad " & 12 & " & 19 & = & \dfrac{12\frac{1}{2}\times 13\times 19}{16\times 12} \end{array}$$

$$\frac{12\frac{1}{2}\times 13\times 19}{16\times 12}=\frac{25\times 13\times 19}{2\times 16\times 12}=\frac{6175}{384}=16\tfrac{31}{384}\text{ hours. } \textit{Ans.}$$

(Ex. 217.)—If 4 men, working 6 days, earn £7 10s., how much will eight men, working 24 days, earn?

$$\begin{array}{lcccl} & \text{Men.} & & \text{Days.} & \\ \text{Wages for} & 4 & \text{working} & 6 & =\text{£}7\ 10\text{s.} \\ \quad " & 1 & " & 6 & \\ \quad " & 1 & " & 1 & \\ \quad " & 8 & " & 1 & \\ \quad " & 8 & " & 24 & \end{array}\left.\begin{array}{c}\\ \\ \\ \\ \\ \end{array}\right\} = \frac{\text{£}7\ 10\text{s.}\times 8\times 24}{4\times 6}$$

$$\frac{\text{£}7\ 10\text{s.}\times \overset{2}{\cancel{8}}\times \overset{4}{\cancel{24}}}{\cancel{4}\times\cancel{6}}=\text{£}60.\ \textit{Ans.}$$

(Ex. 218.)—A field containing 20 acres is to be reaped in 6 days. Suppose a man reaps an acre and a half in two days, how many men must be employed to do the work?

$$\begin{array}{lcccl} & \text{Acres.} & & \text{Days.} & \text{Men.} \\ \text{Men to reap} & 1\frac{1}{2} & \text{in} & 2 & =1 \\ \quad " & 1 & " & 2 & =\dfrac{1}{1\frac{1}{2}} \\ \quad " & 1 & " & 1 & =\dfrac{1\times 2}{1\frac{1}{2}} \\ \quad " & 20 & " & 1 & =\dfrac{1\times 2\times 20}{1\frac{1}{2}} \\ \quad " & 20 & " & 6 & =\dfrac{1\times 2\times 20}{1\frac{1}{2}\times 6} \end{array}$$

$$\frac{1\times 2\times 20}{1\frac{1}{2}\times 6}=\frac{1\times\cancel{2}\times 20\times 2}{3\times\underset{3}{\cancel{6}}}=\frac{40}{9}=4\tfrac{4}{9}\text{ men. } \textit{Ans.}$$

(Ex. 219.)—What will it cost to carry 100 tons 360 miles if two tons be carried 5 miles for 1s. 6d.?

	Tons.		Miles.		Shil.
Carriage of	2	for	5	=	$1\frac{1}{2}$
"	1	"	5		
"	1	"	1	=	$\dfrac{1\frac{1}{2}\times 100\times 360}{2\times 5}$
"	100	"	1		
"	100	"	360		

$$\frac{1\frac{1}{2}\times \overset{50}{\cancel{100}}\times \overset{72}{\cancel{360}}}{\cancel{2}\times \cancel{5}}=(75\times 72)\text{s.}=5400\text{s.}=£270.\quad \textbf{Ans.}$$

(Ex. 220.)—If 13 horses eat 4 quarters of corn in 11 days, how many would eat 12 quarters in 30 days?

	Qrs.		Dys.		Horses.
Horses to eat	4	in	11	=	13
"	1	"	11	=	$\dfrac{13}{4}$
"	1	"	1	=	$\dfrac{13\times 11}{4}$
"	12	"	1	=	$\dfrac{13\times 11\times 12}{4}$
"	12	"	30	=	$\dfrac{13\times 11\times 12}{4\times 30}$

$$\frac{13\times 11\times \overset{3}{\cancel{12}}}{\cancel{4}\times \underset{10}{\cancel{30}}}=\frac{143}{10}=14\tfrac{3}{10}\text{ horses.}\quad \textit{Ans.}$$

(Ex. 221.)—If 8 men and 6 boys do a piece of work in 24 days, how long will it take 12 men and 9 boys to do it, if a man does 3 times as much as a boy?

Bring the men and boys all to boys.

Since 1 man = 3 boys $\therefore$ 8 men = 24 boys, and 24b. + 6b. = 30 boys.

" " $\therefore$ 12 " = 36 boys, and 36b. + 9b. = 45 boys.

(See also Ex. 181.)

$$\begin{array}{lrcl} & \text{Boys.} & & \text{Days.} \\ \text{Time for} & 30 & = & 24 \\ \quad ,, & 1 & = & 24 \times 30 \\ \quad ,, & 45 & = & \dfrac{24 \times 30}{45} \end{array}$$

$$\frac{\overset{8}{\cancel{24}} \times \overset{2}{\cancel{30}}}{\underset{3}{\cancel{45}}} = 16 \text{ days. } \textit{Ans.}$$

(Ex. 222.)—How many girls will be required to make 78 shirts in 8 days, if 32 girls require 6 days to make 36 shirts?

$$\begin{array}{lrcrcl} & \text{Shirts.} & & \text{Days.} & & \text{Girls.} \\ \text{Girls to make} & 36 & \text{in} & 6 & = & 32 \\ \quad ,, & 1 & ,, & 6 & = & \dfrac{32}{36} \\ \quad ,, & 1 & ,, & 1 & = & \dfrac{32 \times 6}{36} \\ \quad ,, & 78 & ,, & 1 & = & \dfrac{32 \times 6 \times 78}{36} \\ \quad ,, & 78 & ,, & 8 & = & \dfrac{32 \times 6 \times 78}{36 \times 8} \end{array}$$

$$\frac{\overset{4}{\cancel{32}} \times \cancel{6} \times \overset{13}{\cancel{78}}}{\underset{6}{\cancel{36}} \times \cancel{8}} = 52 \text{ girls. } \textit{Ans.}$$

(Ex. 223.)—What sum of money must be lent for 42 weeks in order to gain £45, if £658 will gain £60 in 8 weeks?

$$\begin{array}{lrcrcl} & \text{£} & & \text{Weeks.} & & \text{£} \\ \text{Sum lent to gain} & 60 & \text{in} & 8 & = & 658 \\ \quad ,, & 1 & ,, & 8 & & \\ \quad ,, & 1 & ,, & 1 & & \\ \quad ,, & 45 & ,, & 1 & & \\ \quad ,, & 45 & ,, & 42 & & \end{array} \Bigg\} = \frac{£658 \times 8 \times 45}{60 \times 42}$$

$$\frac{£\overset{94}{\cancel{658}} \times \overset{2}{\cancel{8}} \times \overset{3}{\cancel{45}}}{\underset{4}{\cancel{60}} \times \underset{\underset{3}{\cancel{21}}}{\cancel{42}}} = £94. \quad \textit{Ans.}$$

(Ex. 224.)—How long will £56 keep a family of 12 persons, if £32 keep 8 people for 18 days?

	Persons.		£		Days.
Time to keep	8	on	32	=	18
„	1	„	32	=	18×8
„	1	„	1	=	$\frac{18\times 8}{32}$
„	12	„	1	=	$\frac{18\times 8}{32\times 12}$
„	12	„	56	=	$\frac{18\times 8\times 56}{32\times 12}$

$$\frac{\overset{3}{\cancel{18}}\times\cancel{8}\times\overset{14}{\cancel{56}}}{\underset{\cancel{4}}{\cancel{32}}\times\underset{2}{\cancel{12}}}=\frac{42}{2}=21 \text{ days. } \textit{Ans.}$$

(Ex. 225.)—How many men can plough $3\frac{3}{4}$ acres in 9 hours if 6 men plough 5 acres in 8 hours?

	Acres.		Hours.		Men.
Men to plough	5	in	8	=	6
„	1	„	8		
„	1	„	1	=	$\frac{6\times 8\times 3\frac{3}{4}}{5\times 9}$
„	$3\frac{3}{4}$	„	1		
„	$3\frac{3}{4}$	„	9		

$$\frac{6\times 8\times 3\frac{3}{4}}{5\times 9}=\frac{\overset{2}{\cancel{6}}\times\overset{2}{\cancel{8}}\times\overset{\cancel{3}}{\cancel{15}}}{\cancel{5}\times\underset{\cancel{3}}{\cancel{9}}\times\cancel{4}}=2\times 2=4 \text{ men. } \textit{Ans.}$$

INTEREST, INSURANCE.

(Ex. 226.—What is the amount of insurance on £6,968 for 2 years at $4\frac{3}{4}$ per cent.?

	£		Year.		£
Insurance on	100	for	1	=	$4\frac{3}{4}$
„	1	„	1		
„	6968	„	1	=	$\frac{4\frac{3}{4}\times 6968\times 2}{100}$
„	6968	„	2		

$$\frac{£4\frac{3}{4}\times 6968\times 2}{100}=\frac{£19\times\overset{1742}{\cancel{6968}}\times\cancel{2}}{\underset{50}{\cancel{100}}\times\cancel{4}}=\frac{£33098}{50}$$

$=£661$ 19s. $2\frac{1}{4}$d.$+\frac{3}{5}$ (or $2\frac{2}{5}$d.) *Ans.*

(Ex. 227.)—The interest on £475 amounted to £18 14s., what was the rate per cent?

$$\text{Interest on } \overset{\pounds}{475} = \overset{\pounds}{18\tfrac{7}{10}}$$

$$,, \qquad 1 = \frac{18\frac{7}{10}}{475}$$

$$,, \qquad 100 = \frac{18\frac{7}{10} \times 100}{475}$$

$$\pounds\frac{18\frac{7}{10} \times 100}{475} \times \pounds\frac{187 + \overset{\overset{2}{\cancel{10}}}{\cancel{100}}}{\underset{95}{\cancel{475}} \times \cancel{10}} = \pounds\frac{374}{95} = \pounds3\tfrac{89}{95} = \pounds3 \text{ 18s. } 8\tfrac{3}{4}\text{d.} + \tfrac{7}{19}$$

or £3 18s. $8\frac{16}{19}$d. *Ans.*

(Ex. 228.)—What sum will amount to £580 in 4 years at 5 per cent per annum?

Interest on 100 for 4 years at $5\% = \pounds5 \times 4 = \pounds20$

Amount of ,, ,, $= \pounds100 + \pounds20 = \pounds120$

$$\text{Principal giving } \overset{\pounds}{120} = \overset{\pounds}{100}$$

$$,, \qquad 1 = \frac{100}{120}$$

$$,, \qquad 580 = \frac{100 \times 580}{120}$$

$$\frac{\pounds\overset{5}{\cancel{100}} \times \overset{290}{\cancel{580}}}{\underset{\underset{3}{\cancel{6}}}{\cancel{120}}} = \frac{1450}{3} = \pounds483 \text{ 6s. 8d.} \quad \textit{Ans.}$$

(Ex. 229.)—What is the interest on £68 17s. 6d. for 20 years at $2\frac{1}{3}$ per cent?

$$\pounds68 \text{ 17s. 6d.} = \pounds68\tfrac{7}{8}.$$

$$\text{Interest on } \overset{\pounds}{100} \text{ for } \overset{\text{Year}}{1} = \overset{\pounds}{2\tfrac{1}{3}}$$

$$\left.\begin{array}{lll} ,, & 1 & ,, \ 1 \\ ,, & 68\frac{7}{8} & ,, \ 1 \\ ,, & 68\frac{7}{8} & ,, \ 20 \end{array}\right\} = \frac{2\frac{1}{3} \times 68\frac{7}{8} \times 20}{100}$$

$$\pounds\frac{7 \times 551 \times \cancel{20}}{3 \times 8 \times \underset{5}{\cancel{100}}} = \pounds\frac{3857}{120} = \pounds32 \text{ 2s. 10d.} \quad \textit{Ans.}$$

APPENDIX II.*

HOW TO STATE RULE OF THREE SUMS.

BY A GRADUATE OF OXFORD.

Take care of the third and second terms, and leave the first to take care of itself. The third term (or demand) is of the same kind as the required answer.

1st.—SINGLE OR SIMPLE RULE OF THREE.

(a) Example.—If 3lbs. of tea cost 7s., what will 18lbs cost?

BEFORE STATING THE SUM:

1st *Question.*—What is it we want to find in the answer?

Answer.—Money. Therefore 7s. will be the *third* term.

2nd *Question.*—Will 18lb. cost a greater, or a less, sum of money than 3lb.?

Answer.—A greater. Therefore 18lb. must be the *second* term, as it is the *greater* number. The 3lb. becomes the *first* term.

3rd *Question.*—When your sum is stated, what must you do?

Answer.—Multiply the second and third terms together, and divide the product by the first term.

	1st term.		2nd term.		3rd term.
	lbs.		lbs.		Shillings.
As	3	:	18	::	7
			7		
			3)126		
			42s. = £2 2s. *Ans.*		

* By permission of the Rev. J. W. W. Drew, Romford. See Preface.

(b) EXAMPLE.—If 100 cigars cost £1 17s. 6d., how many can you buy for $4\frac{1}{2}$d.?

BEFORE STATING THE SUM :

1st *Question.*—What do we want to find in the answer?

Answer.—Cigars. Therefore 100 cigars must be the *third* term.

2nd *Question.*—Will $4\frac{1}{2}$d. buy a greater, or a less, number of cigars than £1 17s. 6d.?

Answer.—A less. Therefore $4\frac{1}{2}$d. must be the *second* term, as it is the *less* number.

	£ s. d.		d.		Cigars.
As	1 17 6 (=450d.)	:	$4\frac{1}{2}$	::	100

$$\frac{4\frac{1}{2} \times 100}{450} = \frac{450}{450} = 1 \text{ cigar. } \textit{Ans.}$$

2ND.—DOUBLE, OR COMPOUND, RULE OF THREE.

N.B.—The Demand (and not the Supposition) must be regarded as the Question. The Supposition contains three terms, the Demand two only.

(c) EXAMPLE.—If 3 carpenters make 7 boxes in 2 days (supp.), how long will it take 9 carpenters to make 28 boxes? (demand).

BEFORE STATING THE SUM :

1st *Question.*—Will your answer be a number of carpenters, boxes, or days?

Answer.—Days. Therefore 2 days must be the *third* term.

2nd *Question.*—Will 9 carpenters require more, or less, time than 3 to make anything?

Answer.—Less. Therefore 3 is in the *second* term, as it is the *less* number.

3rd Question.—Is more, or less, time required to make 28 boxes than to make 7 boxes?

Answer.—More. Therefore 28 is in the *second* term, as it is the *greater* number.

$$\text{As} \quad \left.\begin{array}{l} 9 \text{ carpenters} \\ \underline{7} \text{ boxes} \\ 63 \end{array}\right\} : \left.\begin{array}{l} 3 \text{ carpenters} \\ \underline{28} \text{ boxes} \\ 84 \end{array}\right\} :: \quad 2 \text{ days.}$$

$$\begin{array}{r} 2 \\ 63)\overline{168}(2 \\ \underline{126} \\ \frac{42}{63} = \frac{2}{3} \end{array} \qquad 2\tfrac{2}{3} \text{ days. } \textit{Ans.}$$

(d) Example.—If 42 men finish a work in 36 days (supp.), how many will finish twice as great a work in 27 days? (dem.)

1st Question.—Your answer will be a number of ——?

Answer.—Men. Therefore 42 men will be the *third* term.

2nd Question.—Will 2 works require a greater, or a less, number of men than 1 work?

Answer.—A greater. Therefore 2 works will be in the *second* term, as it is the *greater* number.

3rd Question.—Would 27 days require a greater, or a less, number of men than 36 days, to do the same work?

Answer.—A greater number, certainly. Therefore 36 days must be in the *second* term, as it is the *greater* number.

$$\text{As} \quad \left.\begin{array}{l} 1 \text{ work} \\ 27 \text{ days} \end{array}\right\} : \left.\begin{array}{l} 2 \text{ works} \\ 36 \text{ days} \end{array}\right\} :: \quad 42 \text{ men.}$$

$$\frac{2 \times 36 \times 42}{1 \times 27} = \frac{3024}{27} = 112 \text{ men. } \textit{Ans.}$$

(e) Example.—If 135 men build 4 houses in 64 days (supp.), how long will it take 60 men to build 1 house? (dem.)

1st Question.—Your answer will be a number of ——?

Answer.—Days. Therefore 64 days will be the *third* term.

2nd Question.—Will 60 men require more, or less, time than 135 men to do the same amount of work?

Answer.—More. Therefore 135 men will be in the *second* term, as it is the *greater* number.

3rd Question.—Will one house require more, or less, time to be spent in building it than 4 houses?

Answer.—Less. Therefore 1 will be in the *second* term, as it is the *less* number.

$$\text{As} \quad \left.\begin{array}{r}60 \text{ men}\\ 4 \text{ houses}\end{array}\right\} : \left.\begin{array}{r}135 \text{ men}\\ 1 \text{ house}\end{array}\right\} :: 64 \text{ days.}$$

$$\frac{135 \times 1 \times 64}{60 \times 4} = \frac{8640}{240} = 36 \text{ days. } \textit{Ans.}$$

NOTE.—The third term is contained in the fourth [*i.e.*, the answer, or number required] as many times as the first is contained in the second. Hence the direct method of working every Rule of Three Sum would be (1) to state it, and then (2) *divide the second term by the first, and multiply the third term by the result.*

EXAMPLE (a)—

	lbs.		lbs.		Shillings.
As	3	:	18	::	7

$$\frac{18}{3} \times 7 = 6 \times 7 = 42 \text{ shillings.}$$

REASON.—7 is contained in 42 as many times as 3 is contained in 18, viz., 6 times, *i.e.*,

As 3lb. : 18lb. :: 7 shillings : 42 shillings.

Or, 3 is to 18 as 1 to 6. In like manner 7 is to 42 as 1 to 6.

EXAMPLE (a)—

	lbs.		lbs.		Shillings.
As	18	:	3	::	42

$$\frac{3}{18} \times 42 = \frac{1}{6} \times 42 = \frac{42}{6} = 7 \text{ shillings.}$$

REASON.—42 is contained in 7 as many times as 18 is contained in 3, viz., $\frac{1}{6}$ time [*i.e.*, $\frac{1}{6}$ of 42 is contained in 7, and $\frac{1}{6}$ of 18 is contained in 3], *i.e.*,

As 18lb. : 3lb. :: 42 shillings : 7 shillings.

Or, 18 is to 3 as 1 to $\frac{1}{6}$. In like manner 42 is to 7 as 1 to $\frac{1}{6}$.

JOHN HEYWOOD, Excelsior Steam Printing and Bookbinding Works,
Hulme Hall Road, Manchester.